# Analysis and Control of UPQC and its DC-Link Power by Use of *p-q-r* Instantaneous Power Theory

Fan Ng[1], Man-Chung Wong[1], Ying-Duo Han[1,2]
[1] Department of Electrical and Electronics Engineering, University of Macau, Macau, SAR, P. R. China
[2] Department of Electrical Engineering, Tsinghua University, Beijing, P. R. China
E-mail: ma36510@umac.mo

**Abstract – This paper provides an analysis and control algorithm of a three-phase four-wire unified power quality conditioner (UPQC) based on *p-q-r* instantaneous power theory. An UPQC is an integration of series and parallel active filters in order to compensate the voltage flicker/imbalance, reactive power, current imbalance, and harmonics. The *p-q-r* theory transforms a three-phase four-wire voltage space vector into a single dc voltage, and the corresponding currents into a dc based active power *p*-axis component and two imaginary power components, *q*-axis and *r*-axis. When *p-q-r* theory is used in reference current control, the *p*-axis current is filtered, and a corresponding *r*-axis component is added to force the reference current onto the $\alpha$-$\beta$ plane. The current space vector follows the movement of voltage space vector's projection on $\alpha$-$\beta$ plane, however, if there are harmonics and negative sequence exists in the voltage, the calculated reference current is not sinusoidal. In this paper an extra *q*-axis component is proposed to add to the original current compensation strategy based on *p-q-r* theory to maintain a sinusoidal current waveform under distorted voltage. With *p-q-r* theory, a control block model of an integration feedback of dc power is proposed to maintain the average dc power to be zero. The pole location of the feedback model and an equivalent model by detecting the instantaneous dc voltage is discussed. The proposed control algorithm is verified by simulation with PSCAD/EMTDC.**

**Keywords – Power quality, UPQC, Active filters, *p-q-r* power theory, inverter dc power.**

## INTRODUCTION

Power electronics loads (nonlinear loads) inject harmonic currents in the ac system and increase overall reactive power demanded by the equivalent load. On the other hand, development in the digital electronics communications and in process control has increased the number of sensitive loads that require ideal sinusoidal supply voltages for their proper operation [1], [2], [6].

To soften the drawbacks produced by the nonlinear loads, unified power quality conditioners (UPQC) were presented during 1998 [2]. The main purpose of a UPQC is to compensate the supply voltage and load current imperfections, such as sags, swells, interruptions, imbalance, flicker, harmonics, reactive currents, and current unbalance. In other words, the UPQC has the capability of improving power quality at the point of installation on power distribution systems or industrial power systems.

A typical configuration of UPQC is shown in Fig. 1. It is a combination of series active filter and parallel active filter. The series active filter is responsible for cancellation of voltage imbalance, flicker, sags and swells, and provides a stable balanced and sinusoidal voltage to the load. The parallel active filter is used to compensate the imbalance, reactive power, neutral current and harmonics of the

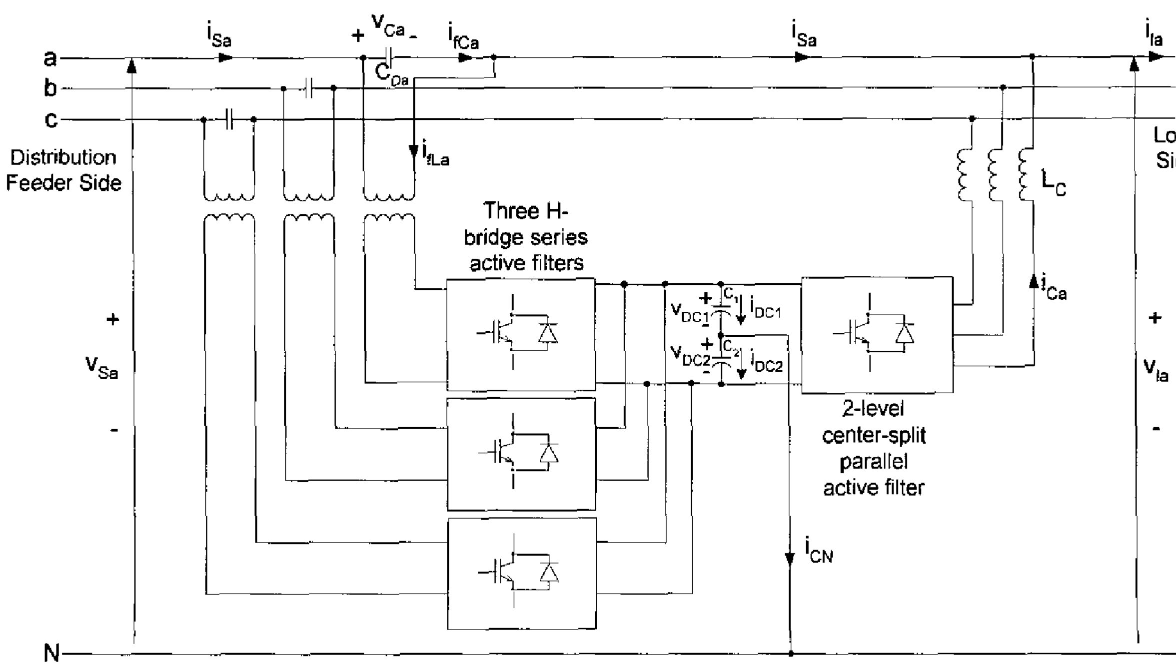


Fig. 1: Configuration of unified power quality conditioner (UPQC) under study

source current. The series active filter topology is referenced from [9], as the three series inverters are connected to the same dc storage capacitors, a coupling transformer is needed to provide isolation.

The UPQC under study in Fig. 1 is analyzed with *p-q-r* instantaneous power theory. The *p-q-r* theory proposed *p-q-r* reference frames which rotates according to the voltage space vector of a three-phase four-wire system [10], [11], [12]. The current space vector is transformed into three linearly independent dc based components, in which *p*-axis component represents the instantaneous active power when multiplied with the single voltage vector component, where the *q*-axis component represents an imaginary current component on $\alpha$-$\beta$ plane, and *r*-axis component represents an imaginary component which is highly related to the 0-axis in $\alpha$-$\beta$-0 reference frames.

A control strategy with the combination of voltage detection method for series active filter and current detection method for parallel active filter is used, with the reference signal determined by the voltage detection and current detection method [10], [12], [14] in *p-q-r* theory.

Since the current is compensated to be a constant length vector that is rotating aligned with the voltage space vector, when the load voltage cannot fully be compensated to be balanced and sinusoidal, the current rotating with the voltage space vector will not be sinusoidal too.

In this paper, an extra *q*-axis component is proposed to add to the reference source current space vector to force it to rotate with a pure sinusoidal reference. The power supplied by the series active filter and loss in switching devices is obtained by drawing extra active power from the parallel active filter. With the aid of *p-q-r* theory, a control block model of the integration feedback is formed and the amount of power is determined by an integration feedback of dc storage power. The equivalent model of integration controller is proposed by detecting the instantaneous value of dc storage capacitor voltage.

The simulation with PSCAD/EMTDC verifies that the proposed analysis and control algorithm is valid for the UPQC under three-phase four-wire system.

## POWER DEFINITION IN $p$-$q$-$r$ REFERENCE FRAME [10], [11], [12]

Voltages and currents in Cartesian $a$-$b$-$c$ coordinates can be transformed to Cartesian $\alpha$-$\beta$-0 coordinates as [3], [4], [5].

$$\begin{bmatrix} e_\alpha \\ e_\beta \\ e_0 \end{bmatrix}_{\alpha\beta 0} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}_{abc} \tag{1}$$

The $p$-$q$-$r$ coordinates are formed by aligning the $\alpha$-axis with the system voltage space vector to form the *p*-axis, thus the *p*-, *q*- and *r*-axis are rotating along with the voltage space vector.

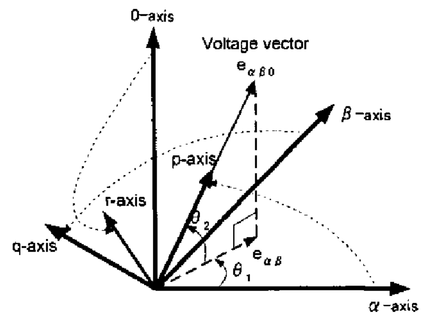


Fig. 2: Rotation of $\alpha$-, $\beta$- and 0-axis to *p*-, *q*- and *r*-axis

The mapping of the $\alpha$-$\beta$-0 coordinates to $p$-$q$-$r$ coordinates is performed by two rotations. First, a new $\alpha'$-$\beta'$-0 reference frames is established by rotating the 0-axis of the $\alpha$-$\beta$-0 reference frames by $\theta_1$, aligning the $\alpha$-axis with the projection line of the voltage space vector to the $\alpha$-$\beta$ plane. The $p$-$q$-$r$ reference frames can be formed by rotating $\beta'$-axis of the $\alpha'$-$\beta'$-0 reference frames by $\theta_2$, aligning the $\alpha'$-axis with the voltage space vector. The $\beta'$-axis is renamed to *q*-axis in the second rotation, and since it is rotated only in the first rotation, the *q*-axis of the $p$-$q$-$r$ reference frames always locates on the surface of the $\alpha$-$\beta$ plane of the $\alpha$-$\beta$-0 reference frames. The transformation from the $\alpha$-$\beta$-0 reference frames to the $p$-$q$-$r$ reference frames can be described as (2), (3), (4) and (5).

$$\begin{bmatrix} i_p \\ i_q \\ i_r \end{bmatrix}_{pqr} = \begin{bmatrix} \frac{e_\alpha}{e_{\alpha\beta 0}} & \frac{e_\beta}{e_{\alpha\beta 0}} & \frac{e_0}{e_{\alpha\beta 0}} \\ -\frac{e_\beta}{e_{\alpha\beta}} & \frac{e_\alpha}{e_{\alpha\beta}} & 0 \\ -\frac{e_0 e_\alpha}{e_{\alpha\beta} e_{\alpha\beta 0}} & -\frac{e_\beta e_0}{e_{\alpha\beta} e_{\alpha\beta 0}} & \frac{e_{\alpha\beta}}{e_{\alpha\beta 0}} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix}_{\alpha\beta 0} \tag{2}$$

$$\begin{bmatrix} e_p \\ e_q \\ e_r \end{bmatrix}_{pqr} = \begin{bmatrix} e_{\alpha\beta 0} \\ 0 \\ 0 \end{bmatrix}_{pqr} \tag{3}$$

where

$$e_{\alpha\beta} = \sqrt{e_\alpha^2 + e_\beta^2} \tag{4}$$

$$e_{\alpha\beta 0} = \sqrt{e_\alpha^2 + e_\beta^2 + e_0^2} = \sqrt{e_a^2 + e_b^2 + e_c^2} \tag{5}$$

The instantaneous active power *p* is defined as a dot product of the voltage and current as [3], [4], [5], [10], [11], [12]

$$p \equiv \vec{e} \cdot \vec{i} = e_p i_p \tag{6}$$

Instantaneous imaginary power *q* is defined as a cross product of the voltage and current vectors as [3], [4], [5], [10], [12]

$$\vec{q} \equiv \vec{e} \times \vec{i} = \begin{bmatrix} 0 \\ -e_p i_r \\ e_p i_q \end{bmatrix}_{pqr} \equiv \begin{bmatrix} 0 \\ q_q \\ q_r \end{bmatrix}_{pqr} \tag{7}$$

With the $p$-$q$-$r$ theory, the concept of instantaneous imaginary power is clear. The current vector components $i_q$ and $i_r$ are the current components that are orthogonal to the voltage vector, and thus do not contribute to the instantaneous active power. The $q$-axis component of the current vector is always located on the $\alpha$-$\beta$ plane where the $r$-axis component is highly related to the neutral line current.

For a balanced and sinusoidal voltage vector, the value of $e_p$ is constant and thus the $p$-$q$-$r$ transformation can be viewed as a transformation from three-phase periodic system to three separated dc systems. If the angular velocity of the voltage vector on $\alpha$-$\beta$ plane is constant, a constant value of current vector represents a balanced and sinusoidal three-phase current.

## Instantaneous Power Flow Analysis of a UPQC with $p$-$q$-$r$ Theory

The reference load voltage of a UPQC system is taken to be balanced and sinusoidal as for the requirement of sensitive loads, however, the fundamental phase angle between the source voltage and load voltage can be varied, and so as the source current's phase angle with respect to the load voltage. Varying the phase angles results approximately same amount of power supplied by the source, however, the magnitude of the source current and the magnitude of series inverter voltage varies. In this section, the instantaneous power flow in a UPQC system is analyzed such that the source current and the series active filter voltage are kept as minimum as possible, which can reserve a larger capacity of power supplied to the load and a larger ability to solve voltage flicker by the series active filter.

### A. *Instantaneous Active Power Flow in p-Frame*

The instantaneous active power is the dot product of the voltage and current space vector. In the case of $p$-$q$-$r$ frames, the dot product results the scalar product of the voltage magnitude and the $p$-axis current only. Consider the UPQC system shown in Fig. 1, the series active filter and parallel active filter can control the load voltage $\vec{v}_l$ and the source current $\vec{i}_S$ respectively. For the load, the UPQC is responsible to maintain a balance and sinusoidal rated voltage. The instantaneous active power loops in the UPQC system and imaginary power provided by the source should be reduced to a smaller value such that a larger capacity of power is available to the load side. The instantaneous active power flow of the UPQC system is shown in Fig. 3, in which only phase a series inverter and electrical quantities are shown.

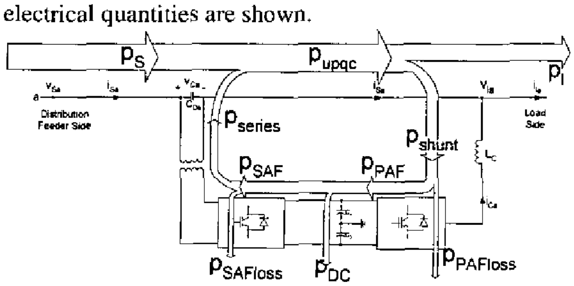


Fig. 3: Instantaneous active power flow in the UPQC system

The parallel active filter power loss $p_{PAFloss}$ is the switching loss inside the parallel inverter. The series active filter power loss $p_{SAFloss}$ includes the switching loss in the inverters and the cooper loss in the coupling transformers. When there is voltage flicker, sag, or the voltage is below the rated voltage, ac and dc active power is needed to draw from the series inverter to the load, and this power is named $p_{series}$. The power $p_{shunt}$ has an ac component which is used to compensate harmonics and imbalance in the load current in which will increase the ac portion of $p_{upqc}$ and $p_S$ if remain uncompensated. The dc component of $p_{shunt}$ is used to supply power for $p_{series}$, $p_{loss}$, such that the average of the dc storage power $p_{dc}$ equals zero. Since the power $p_{series}$ is supplied from the UPQC to the load side and $p_{shunt}$ is taken from the load side, it is clear that there is an amount of active power looping inside the UPQC system and they have the relationship related to the dc capacitor power $p_{dc}$ and the loss power $p_{loss}$ as stated in (8).

$$p_{shunt} = p_{series} + p_{dc} + p_{loss} \tag{8}$$

where

$$p_{loss} = p_{SAFloss} + p_{PAFloss} \tag{9}$$

Notice that in (8) the power relationship inside a UPQC system excludes the source and the load active powers. This implies that the amount of power looping inside the UPQC is not determined by the source to load transmission.

The instantaneous active power flow relationship between the source and load can be defined by (10).

$$p_S = p_l + p_{loss} + p_{DC} \tag{10}$$

Source active power is supplied to the load and also to cover the loss power and the dc power, however, they have no relationship with the amount of power looping in the UPQC.

### B. *Instantaneous Imaginary Power Flow in q- and r-Frame*

The vector of instantaneous imaginary power vector is the cross product of the voltage and current space vector, in which, when multiply with the voltage, the $q$- and $r$-axis current forms the $r$- and $q$-axis imaginary power respectively. If there are $q$- and $r$-axis current components, or a non-zero vector of imaginary power, the capacity of active power transferable from the source to the load is reduced. However, the parallel and series active filter can provide to compensate the imaginary power such that the capacity used other than supply in active power is minimum.

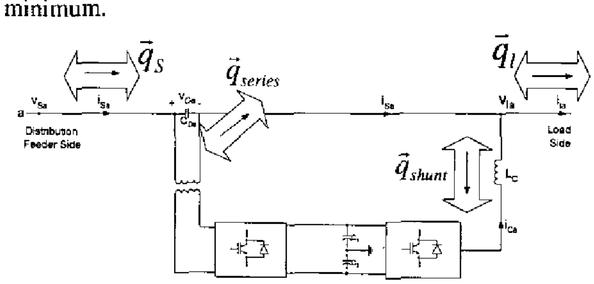


Fig. 4: Instantaneous imaginary power flow in the UPQC system

The instantaneous imaginary power flow of the UPQC system is shown in Fig. 4. The series and parallel active filters can supply required amount of $\vec{q}_{series}$ and $\vec{q}_{shunt}$ as long as the capacity of the active filters is enough. A non-zero average vector sum of the instantaneous imaginary power is a result of phase difference between the voltage and current. In the point of view of voltage and current space vectors, it is formed if there is an average deviation in the difference of direction of voltage and current space vectors. By the *p-q-r* theory, a non-zero average vector sum of instantaneous imaginary power, i.e. the average of $q_q$ and $q_r$ are not zero, is formed when there is a dc value in the *q*-axis and *r*-axis which are orthogonal to the voltage space vector. The dc component in *r*-axis imaginary power refers to the reactive power component in the load, and the ac component is related to the harmonics and imbalance. The *q*-axis imaginary power is mainly produced by the neutral current imbalance. Reducing the amount of imaginary power supplied by the source results an increase of capacity of active power available to the load side. The imaginary power $\vec{q}_l$ can be fully compensated by the parallel active filter while the series active filter supplies an imaginary power with a zero average vector sum to compensate the imbalance and harmonics in the source voltage.

## Compensation of Instantaneous Active and Imaginary Powers

To increase the capacity of power available to the load, the ac component of load power $\tilde{p}_l$ , and $\vec{q}_l$ , should be compensated by the parallel active filter, the average of series active filter power $\overline{p}_{series}$ , and $\vec{q}_{series}$ , should be considered such that the source current magnitude is as small as possible.
Table 1 shows a summary of components of instantaneous active and reactive power which can be altered by the selection of references or compensated by the UPQC system. $\tilde{p}_l$ represents the ac component of the load instantaneous active power, $\overline{q}_{lq}$ represents the average value of *q*-axis imaginary power of the load, and $\overline{\vec{q}_S} = \begin{bmatrix} 0 & \overline{q}_{Sq} & \overline{q}_{Sr} \end{bmatrix}^T_{pqr}$ is named as the average vector sum of source imaginary power vector, and the same for other quantities. Some of the main reasons of existence are made under the assumption that the load voltage and source current are compensated to be balanced and sinusoidal.

**Table 1: Summary of components of instantaneous power in the system**

| Parameters | Main reasons of existence | Compensation or elimination |
|---|---|---|
| $\tilde{p}_l$ | Exist if there are load voltage and current imperfections | Load voltage compensated by SAF. Harmonics and imbalance in current compensated by PAF |
| $\overline{q}_{lr}$ | Phase difference between fundamental positive sequence load voltage and load current | Compensation by PAF |
| $\tilde{q}_{lr}$ | Harmonics and imbalance in current | Compensation by PAF |
| $q_{lq}$ | Harmonics and imbalance in load, mainly neutral line current | Compensation by PAF |
| $\overline{p}_{series}$ | Source voltage deviation from the rated voltage, phase difference between fundamental source voltage and load voltage | Reduced by consideration of reference load voltage placement |
| $\overline{\vec{q}}_{series}$, $\overline{\vec{q}}_S$ | Phase difference between fundamental source voltage and load voltage | Compensated by consideration of reference load voltage placement |
| $\tilde{p}_S$ and $\vec{\tilde{q}}_S$ | Harmonics and imbalance in source voltage | Compensated by SAF with $\overline{p}_{series}$, $\vec{\tilde{q}}_{series}$ |

If the voltage vectors $\vec{v}_l$, $\vec{v}_C$, $\vec{v}_S$ and current vectors $\vec{i}_S$, $\vec{i}_C$, $\vec{i}_l$ are represented by a *p-q-r* reference frames rotating with $\vec{v}_l$ , the vectors and instantaneous active power expression can be given by (11) to (20).

$$\vec{v}_l = \begin{bmatrix} v_{lp} & 0 & 0 \end{bmatrix}^T_{pqr(\vec{v}_l)} \tag{11}$$

$$\vec{v}_C = \begin{bmatrix} v_{Cp} & v_{Cq} & v_{Cr} \end{bmatrix}^T_{pqr(\vec{v}_l)} \tag{12}$$

$$\vec{v}_S = \begin{bmatrix} v_{Sp} & v_{Sq} & v_{Sr} \end{bmatrix}^T_{pqr(\vec{v}_l)} \tag{13}$$

$$\vec{i}_S = \begin{bmatrix} i_{Sp} & i_{Sq} & i_{Sr} \end{bmatrix}^T_{pqr(\vec{v}_l)} \tag{14}$$

$$\vec{i}_C = \begin{bmatrix} i_{Cp} & i_{Cq} & i_{Cr} \end{bmatrix}^T_{pqr(\vec{v}_l)} \tag{15}$$

$$\vec{i}_l = \begin{bmatrix} i_{lp} & i_{lq} & i_{lr} \end{bmatrix}^T_{pqr(\vec{v}_l)} \tag{16}$$

$$p_l = v_{lp} i_{lp} \tag{17}$$

$$p_{shunt} = -v_{lp} i_{Cp} \tag{18}$$

$$p_{upqc} = v_{lp} i_{Sp} \tag{19}$$

$$p_{series} = -\left(v_{Cp} i_{Sp} + v_{Cq} i_{Sq} + v_{Cr} i_{Sr}\right) \tag{20}$$

The power $p_{upqc}$ includes the load power $p_l$, loss power $p_{loss}$ and also the power supplied by the series inverter $p_{series}$. This means the capacity of power available to the load side is reduced if the power $p_{series}$ is increased. The load power and loss power are unknowns, however, from (19) the magnitude of source current can be minimized if $\vec{i}_S = \begin{bmatrix} i_{Sp} & 0 & 0 \end{bmatrix}^T_{pqr(\vec{v}_l)}$ , since the value of $v_{lp}$ should be compensated to be a constant by the series active filter. This means the current should be compensated to be balanced, sinusoidal and in phase with $\vec{v}_l$ when all the harmonic components of the load current are compensated by the parallel active filter.
If the source current equals $\begin{bmatrix} i_{Sp} & 0 & 0 \end{bmatrix}^T_{pqr(\vec{v}_l)}$ , the instantaneous active power relationship in (20) and the instantaneous imaginary power relationships can be given as (21) to (24).

$$p_{series} = -v_{Cp} i_{Sp} \quad (21)$$

$$\vec{q}_{series} = \begin{bmatrix} 0 \\ q_{series,q} \\ q_{series,r} \end{bmatrix} = \begin{bmatrix} 0 \\ -v_{Cr} i_{Sp} \\ +v_{Cq} i_{Sp} \end{bmatrix}_{pqr(\vec{v}_l^*)} \quad (22)$$

$$\vec{q}_S = \begin{bmatrix} 0 \\ q_{S,q} \\ q_{S,r} \end{bmatrix} = \begin{bmatrix} 0 \\ v_{Sr} i_{Sp} \\ -v_{Sq} i_{Sp} \end{bmatrix}_{pqr(\vec{v}_l^*)} \quad (23)$$

$$\vec{q}_{upqc} = \vec{v}_l \times \vec{i}_S = \vec{O}_{pqr(\vec{v}_l^*)} \quad (24)$$

Consider the load voltage is balanced and sinusoidal, the value of $i_{Sp}$ is constant, in (22), the average vector sum of the instantaneous imaginary power of $\vec{q}_{series}$ is zero when the value of $v_{Cq}$ and $v_{Cr}$ has no dc components, which is the same case for $v_{Sq}$ and $v_{Sr}$ in (23). Since the series active filter voltage is the difference between the load voltage and source voltage, the value of $v_{Cq}$ and $v_{Cr}$, so as $v_{Sq}$ and $v_{Sr}$ has no dc components only if the fundamental positive sequqnce component of the source voltage is in phase with the reference load voltage. A vector diagram showing the relationship of fundamental positive sequence vector in *p-q-r* frame $\vec{v}_l$, $\vec{v}_C$ and $\vec{v}_S$, and the *p*-axis component of $\vec{v}_C$ is shown in Fig. 5.

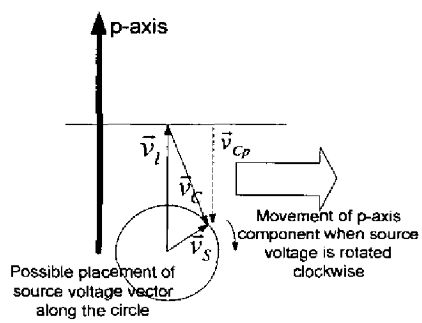


Fig. 5: Vector diagram showing dc component vector in *p-q-r* coordinates of $\vec{v}_l$, $\vec{v}_C$ and $\vec{v}_S$

In (21), minimizing the value of $v_{Cp}$ results reduction of active power drawn from the dc link by the series active filter. In Fig. 5, the vector $\vec{v}_C$ is different from the *p*-axis component vector when the fundamental positive sequence of source voltage is not aligned with the reference load voltage, and the minimum value of $v_{Cp}$ is obtained when the fundamental positive sequence of $\vec{v}_S$ is aligned with the *p*-axis. In this case, the value of $v_{Cp}$ reaches the minimum point, $|\vec{v}_C|=v_{Cp}$, $\vec{q}_{series}=0$ and $\vec{q}_S=0$. To achieve this, the reference load voltage is taken in phase with the fundamental positive sequence of the source voltage.

Assuming that the source voltage is balanced and sinusoidal but is below the rated voltage, the minimum magnitude of $\vec{v}_C$ occurs when $\vec{v}_S$ is in phase with $\vec{v}_l$. In this case, $\vec{v}_C = \begin{bmatrix} v_{Cp} & 0 & 0 \end{bmatrix}^T_{pqr(\vec{v}_l)}$. However, alternating $v_{Cq}$ and $v_{Cr}$ are needed when the source voltage vector is not ideal. The reference load voltage $\vec{v}_l^*$ is calculated by the algorithm which is based on PQR instantaneous power theory in [13], [14].

In order to provide a maximum available capacity for the load side, the power drawn by the series active filter from the dc link can be reduced by selecting the reference load voltage to be in phase with the fundamental positive sequence of source voltage, and the current reference is selected to eliminate the load current's harmonics, imbalance and reactive power, and includes an active power for the power given out by the series active filter. The control algorithms obtaining the corresponding references are discussed in the following section.

## CONTROL ALGORITHM

To obtain the voltage and current references as analyzed above, fundamental positive sequence of source voltage is needed to be extracted, and the load current components corresponding to the harmonics, imbalance and reactive power must be extracted. A block diagram that shows the procedure to calculate the voltage reference, current reference and the determination of dc component of instantaneous active power absorbed by the parallel active filter, namely $p_{comp}$.

To extract the fundamental positive sequence of the source voltage, the reference wave generator proposed in [13], [14] is applied since a balanced and sinusoidal unity reference wave can be calculated without time delay. The current compensation based on *p-q-r* theory in [12] is applied because no voltage preprocessing is needed. An

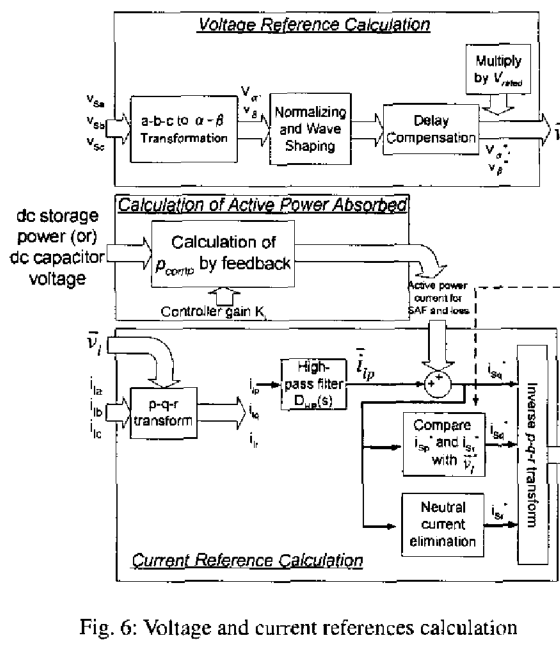


Fig. 6: Voltage and current references calculation

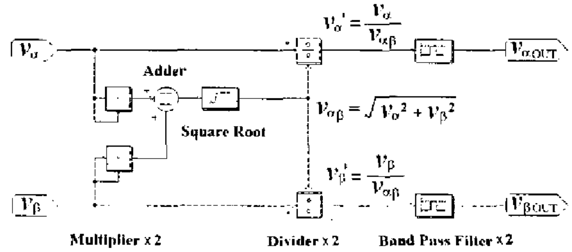


Fig. 7: Normalizing and wave shaping in reference wave generator in [14]

extra $q$-axis component is proposed to add to the reference current when the load voltage cannot be fully compensated, which will force the reference current space vector aligns with the balanced and sinusoidal reference wave. The calculation of the power needed to be absorbed by the parallel active filter, $p_{comp}$, is performed by detecting the dc storage power and feedback with an integration controller, or, if the dc storage is a capacitor, detecting the instantaneous value of capacitor voltage.

### A. Voltage Reference Calculation

The reference wave for the voltage reference can be generated by the reference wave generator (RWG) proposed in [13], [14]. By this algorithm, a balanced and sinusoidal reference wave can be obtained without time delay, and can be maintained even if the source voltage is distorted by severe voltage sags. The block diagram of the RWG is shown in Fig. 7, since the series active filter is controlled by a feedback system with the load voltage on each phase as in [15], the compensating voltage $\vec{v}_C^*$ is not required to be calculated.

### B. Current Reference Calculation

For the reference current determination, the load voltage $\vec{v}_l$ is taken as the $p$-$q$-$r$ rotation reference instead of the reference load voltage $\vec{v}_l^*$ since this guarantees the parallel active filter to compensate the load current harmonics and to provide the correct amount of power looping in the UPQC, even if the series active filter cannot fully compensate for the harmonics, unbalance and voltage sag occurs in the source voltage. Since the $p$-$q$-$r$ theory does not require voltage preprocessing [12], the control algorithm still works under this case. Reference current control in [12] is used since only the harmonics in the current is filtered, in which the parallel active filter can only control the source current. Unlike a conventional parallel active filter, it should absorb (or release when the source voltage is higher than the rated voltage) active power to provide the proper operation of series active filter, such that the average of the dc capacitor voltage $\bar{p}_{DC} = 0$.
The current harmonics and neutral line current can be compensated by the following reference source current vector as in [12].

$$\vec{i}_{Spr}^* = \begin{bmatrix} i_{Sp}^* \\ 0 \\ -\frac{v_{l0}}{v_{l\alpha\beta}} i_{Sp}^* \end{bmatrix}_{pqr(\vec{v}_l)} = \begin{bmatrix} \bar{i}_{lp} + i_{comp} \\ 0 \\ -\frac{v_{l0}}{v_{l\alpha\beta}} i_{Sp}^* \end{bmatrix}_{pqr(\vec{v}_l)} \qquad (25)$$

The dc component of $i_{lp}$, which corresponds to the average active power component of the load current, is extracted, and a small amount of $r$-axis current is added to compensate the neutral current. The current $i_{comp}$ is added to provide compensation of series active filter power and loss power, such that $p_{comp} = v_l\, i_{comp}$. However, when there are harmonics and negative sequence components in the load voltage, the $p$-$q$-$r$ reference frames are not rotating at a constant angular velocity and thus the resulting source current is not sinusoidal.
Here, a compensation method is proposed to control the reference current space vector to follow the rotation of the sinusoidal reference wave. Since the $p$-axis and $r$-axis are used to compensate the active power harmonics and neutral line currents respectively, and the remaining $q$-axis is always located on the $\alpha$-$\beta$ plane, it can be used to move the current space vector to be aligned with the reference wave, without using extra active power. The sinusoidal reference wave $\vec{v}_l^*$ generated by RWG can be transformed into the $p$-$q$-$r$($\vec{v}_l^*$) reference frames and compare with $\vec{i}_{Spr}^*$. The corresponding vector diagram is shown in Fig. 8.

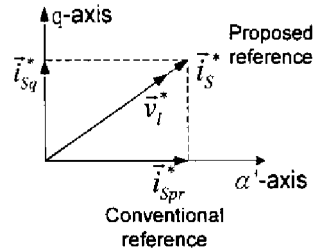


Fig. 8: Vector diagram for current control when $\vec{v}_l^*$ and $\vec{v}_l$ are not aligned

The $\alpha$'-axis is the projection of the $p$-axis on $\alpha$-$\beta$ plane, and is also the direction of the conventional reference current control in $p$-$q$-$r$ theory. However, when the voltage is not balanced and sinusoidal, the instantaneous frequency of rotation of the $p$-$q$-$r$ reference frames is not constant, thus there is a deviation between the current reference and the ideal voltage reference. The $q$-axis component is used to force the original current vector $\vec{i}_{Spr}^*$ to be aligned with the balanced and sinusoidal reference wave. Notice that no extra active power is needed since the $q$-axis component is orthogonal to the load voltage and thus is related to the instantaneous imaginary power only. The value of $i_{Sq}^*$ can be calculated by (26).

$$i_{Sq}^* = \frac{\begin{bmatrix} v_{l\alpha} \\ v_{l\beta} \end{bmatrix} \times \begin{bmatrix} v_{l\alpha}^* \\ v_{l\beta}^* \end{bmatrix}}{\begin{bmatrix} v_{l\alpha} \\ v_{l\beta} \end{bmatrix} \cdot \begin{bmatrix} v_{l\alpha}^* \\ v_{l\beta}^* \end{bmatrix}} \frac{v_{lp}}{v_{l\alpha\beta}} i_{Sp}^* \qquad (26)$$

Although the magnitude of the reference current vector is now not equal to the conventional one which is constant, for a small difference between $\vec{i}_{Spr}^*$ and $\vec{v}_l^*$, the magnitude of $i_{Sq}$ is small compare with $\left|\vec{i}_{Spr}^*\right|$, such that $\left|\vec{i}_S^*\right| \approx \left|\vec{i}_{Spr}^*\right|$.
Subtracting the load current with the reference source current, the reference compensating current $i_{Cp}^*$, $i_{Cq}^*$ and $i_{Cr}^*$ can be defined as (27) to (30).

$$i_{Sp}^* = \bar{i}_{lp} + i_{comp} \qquad (27)$$

$$i_{Cp}^* = i_{lp} - i_{Sp}^* = \tilde{i}_{lp} - i_{comp} \qquad (28)$$

$$i_{Cq}^* = i_{lq} - \frac{\begin{bmatrix} v_{l\alpha} \\ v_{l\beta} \end{bmatrix} \times \begin{bmatrix} v_{l\alpha}^* \\ v_{l\beta}^* \end{bmatrix}}{\begin{bmatrix} v_{l\alpha} \\ v_{l\beta} \end{bmatrix} \cdot \begin{bmatrix} v_{l\alpha}^* \\ v_{l\beta}^* \end{bmatrix}} \frac{v_{lp}}{v_{l\alpha\beta}} i_{Sp}^* \qquad (29)$$

$$i_{Cr}^* = i_{lr} - \frac{v_{l0}}{v_{l\alpha\beta}} i_{Sp}^* \qquad (30)$$

A certain amount of dc current $i_{comp}$ is drawn from the source in order to provide active power such that the series active filter can provide a rated sinusoidal voltage and the losses can be compensated. If the average instantaneous frequency of the rotating *p-q-r* frames equals to that of the reference sinusoidal wave, there will only be an ac component exists in the *q*-axis reference, and thus no dc reactive power is needed from the source.

In this section, a *q*-axis component is introduced to add to the conventional *p-q-r* current compensation in [12] such that the rotation of the current space vector is at a constant instantaneous frequency. The amount of dc current $i_{comp}$ needed to be drawn from the shunt active filter to the dc link is discussed in next section.

### C. Calculation of Active Power Needed to be Absorbed in UPQC

To maintain the dc capacitor voltage to be a constant value, a dc value of active power in $p_{shunt}$ is needed to be drawn from the load side into the UPQC system such that it is equal to the summation of $p_{series}$ and $p_{loss}$ in average. Although $p_{series}$ can be monitored by observing $\vec{v}_C$ and $\vec{i}_S$, the loss power is difficult to be calculated and is related to external variables such as temperature, pressure, etc.

With the *p-q-r* transform, together with the assumption that the load voltage is fully compensated to be the same as $v_l^*$, and the source current $\vec{i}_S$ is compensated to be in phase with $v_l^*$, with the simple active power relationships in (17), (18), (19), (21), a control block diagram is drawn with $p_l$, $p_{comp}$ and $p_{loss}$ as inputs and $p_{DC}$ as output, where $p_{comp} = v_{lp}\, i_{comp}$.

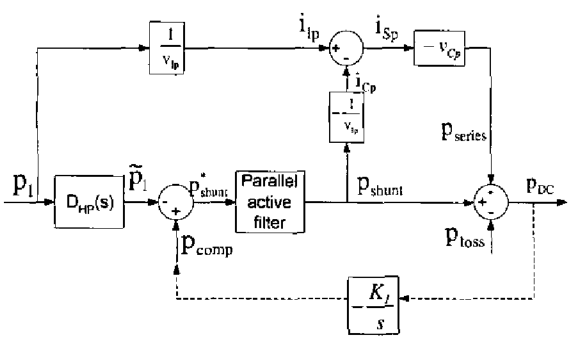


Fig. 9: Control block diagram under the assumption $\vec{i}_S = \begin{bmatrix} i_{Sp} & 0 & 0 \end{bmatrix}^T_{pqr(\vec{v}_l)}$

In Fig. 9, the value of $v_{lp}$ should be compensated to be constant, while $v_{Cp}$ is a varying coefficient when the source voltage is not balanced and sinusoidal. If the parallel active filter can output the calculated reference current without error, it can be viewed as a unity gain block in the diagram. In order to make the steady-state value of average dc power to be zero, a negative integration feedback with gain $K_I$ is added.

The expression of Laplace transform of the dc capacitor power and the shunt active power are shown in (31) and (32).

$$P_{DC}(s) = \frac{s\left[\left(\frac{v_{Cp}}{v_{lp}} - \left(1+\frac{v_{Cp}}{v_{lp}}\right)D_{HP}(s)\right)P_l(s) - P_{loss}(s)\right]}{s+K_I\left(1+\frac{v_{Cp}}{v_{lp}}\right)} \tag{31}$$

$$P_{shunt}(s) = \frac{\frac{v_{Cp}}{v_{lp}}K_I}{s+K_I\left(1+\frac{v_{Cp}}{v_{lp}}\right)}P_l(s) - \frac{s}{s+K_I\left(1+\frac{v_{Cp}}{v_{lp}}\right)}D_{HP}(s)P_l(s) + \frac{K_I}{s+K_I\left(1+\frac{v_{Cp}}{v_{lp}}\right)}P_{loss}(s) \tag{32}$$

From the above equations, there is an extra pole added to the closed-loop system. The location of the pole is given as below.

$$s_{pole} = -K_I\left(1+\frac{v_{Cp}}{v_{lp}}\right) \tag{33}$$

The system will become unstable if $v_{lp} \le -v_{Cp}$, which is reasonable, because in the equality case there is no power released by the source side. In (31) the closed-loop system results a high-pass filtering for the dc capacitor power, and thus the average $p_{DC}$ equals zero in steady-state. Although increasing the value of $K_I$ can increase the stability margin, from (32), the resulting high-pass function is applied to the original filtered harmonics of the load power, if the pole is too far from the origin, high frequency components of the load current will be filtered out and thus limiting the effect of harmonics compensation by the parallel active filter. Thus, a constraint of the value of $K_I$ is given by (34).

$$K_I \le \frac{\omega_{D_{HP}(s)cut-off}}{\left(1+\frac{\left|v_{Cpmax}\right|}{v_{lp}}\right)} \tag{34}$$

Since the dc power in voltage source PWM inverters are switching at high frequency, it is difficult to perform numerical integration by DSPs, in which there will be a high degree of error if the sampling frequency to obtain the dc power signal is low. However, for the dc storage capacitor model used in this UPQC system, the integration of the dc power can be replaced by the detection of instantaneous dc voltage value.

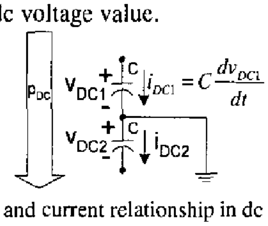


Fig. 10: Voltage and current relationship in dc storage capacitors

The equivalent of the integration controller can be given as (35)

$$-K_I\int_{t_0}^{t} p_{DC}(\tau)d\tau = \frac{K_I C}{2}[v_{DC1}^2(t_0)+v_{DC2}^2(t_0) - v_{DC1}^2(t) - v_{DC2}^2(t)] \tag{35}$$

If the initial value of dc capacitor value is $v_{DC1}(t_0) = v_{DC2}(t_0) = \frac{1}{2}V_{DC0}$, (35) becomes

$$-K_I\int_{t_0}^{t} p_{DC}(\tau)d\tau = \frac{K_I C}{4}V_{DC0}^2 - \frac{K_I C}{2}\left(v_{DC1}^2(t)+v_{DC2}^2(t)\right) \tag{36}$$

From (36) it is clear that the sum of the steady-state value of $v_{DC1}$ and $v_{DC2}$ is lower than that of $V_{DC0}$ if the average of $p_{shunt}$ is not zero. In the case of balanced dc capacitor voltages, i.e. $v_{DC1}(t)=v_{DC2}(t)$ , the capacitor voltage at steady state is given as below.

$$v_{DC1}(t\to\infty)=\sqrt{\frac{1}{4}V_{DC0}^{2}-\frac{4}{K_{I}C}\bar{p}_{shunt}} \tag{37}$$

A larger value of $K_I$ reduces the deviation of the steady-state voltage to the initial voltage, thus, this can be set as a constraint of the value of $K_I$, together with (34), and the range of $K_I$ is given by (38).

$$\frac{4a\,\bar{p}_{shunt\max}}{C\left(\frac{1}{4}V_{DC0}^{2}-v_{DC1\min}^{2}\right)}<K_{I}\le\frac{\omega_{D_{HP}(s)cut-off}}{\left(1+\frac{\left|v_{Cp\max}\right|}{v_{lp}}\right)} \tag{38}$$

where $a$ is a coefficient larger than 1 to provide a safety margin. The value of $V_{DC0}$ can also be adjusted such that the dc storage capacitor voltage does not drop below the requirements of the inverters.

With *p-q-r* theory, a simple active power relationship is obtained and thus a control block model is proposed. An integration controller with gain $K_I$ is used to determine the amount of active power needed for the dc link to provide compensation of series active filter and losses. From the Laplace transform relationship there is a resultant high-pass function applied on the dc-link power, and thus its steady-state power is zero. However, this high-pass function is also applied on the instantaneous active power of the parallel active filter, which will limit the harmonics compensation of the source current, based on this effect a constraint of $K_I$ is defined. The equivalent of this integration by detecting the dc voltage instantaneously is proposed, which can be applied to dc link structure that has equivalent single or single-split capacitor model. The steady-state value of the dc capacitor voltage is investigated and the lower boundary of the gain $K_I$ is set.

## SIMULATION RESULTS AND DISCUSSION

Table 2 shows the values of parameters that are used in simulation with PSCAD/EMTDC for the UPQC shown in Fig. 1. The source is selected to be a highly harmonically distorted unbalanced voltage source and the waveform is shown in Fig. 11(a), and the load is three single phase rectifier which will drive unbalanced and nonlinear currents from the UPQC.

**Table 2: Parameters used in simulation**

| System | Parameters | Value |
|---|---|---|
| Series active filter | Filter capacitor $C_D$ | 11 $\mu$F |
| | Coupling transformer capacity | 3.3kVA |
| | Coupling transformer leakage reactance | 10% |
| | Coupling transformer rated voltage (inverter side/filter capacitor side) | (440V/220V) |
| | PWM Control | SPWM |
| Parallel active filter | Inductance $L_C$ | 20mH |
| | PWM Control | Hysteresis |
| UPQC | dc capacitor $C_1$, $C_2$ | 3300 $\mu$F each |
| | Initial dc voltage $V_{DC0}$ | 800V |
| | Integral feedback gain $K_I$ | 50 |

### A. *Compensation of current under highly distorted source voltage*

When the source voltage is highly distorted by harmonics as in Fig. 11(a), the harmonics in the load voltage in (b) cannot completely be removed by the series active filter, thus the waveform of the load voltage is not completely sinusoidal.

When the current is compensated by taking the reference source current as in (25), in which the *q*-axis reference source current equals zero, and *r*-axis component is used to compensate the neutral line current. The current waveform is plotted in Fig. 12.

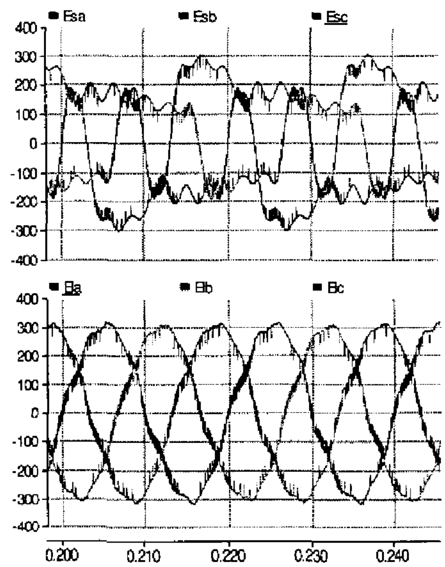


Fig. 11: Waveform of a) highly distorted source voltage, b) compensated load voltage

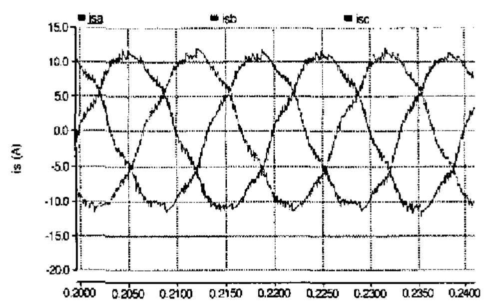


Fig. 12: Current waveform when taking the reference as (25)

It is shown that the current waveform is similar to the load voltage, which is close to the sinusoidal reference load voltage. Using the reference load voltage wave and the *q*-axis source current as in (26), the source current can be compensated to sinusoidal as shown in Fig. 13.

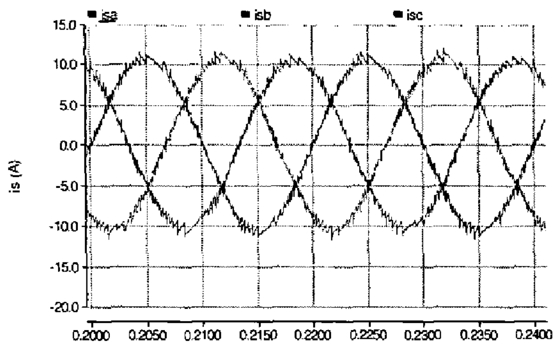


Fig. 13: Current waveform when taking the compensating current reference as (27) to (30)

The simulation result verifies that if there is a balanced sinusoidal reference, with the *p-q-r* theory the source current can be compensated to be balanced and sinusoidal even if the load voltage is not perfectly compensated.

*B. Integration feedback control of dc power*

Three single phase rectifier is act as a nonlinear load for the simulation. The load absorbs an average active power of approximately 2.5kW, and there are 3rd and 5th harmonics exist in the source voltage. Throughout a 0.5 sec simulation, a single-line-to-ground fault occurs in phase *a* at the source side in 0.20s and it is cleared at 0.35s. The waveforms of the source voltages are shown in Fig. 14.

The UPQC starts its operation at 0.10s, in which the load voltage is compensated to be balanced and sinusoidal after that, as shown in Fig. 15. The current waveforms of the load are shown in Fig. 16.

The waveforms of source currents are shown in Fig. 17. Notice that the current is larger during fault in order to provide active power for the series active filter to compensate the nearly zero voltage at phase *a*.

The waveforms of voltages and currents in *p-q-r* reference frames are shown in Fig. 18. Since the load voltage is compensated to be balanced and sinusoidal, the load voltage in the *p-q-r* frames with respect to itself is a constant value. Also, the *q*- and *r*-axis of the source current is compensated to zero, with a dc value in *p*-axis, this shows that the current is compensated to be balanced and sinusoidal. Notice that the source voltage have no dc components in the *q*-axis and *r*-axis, and since the current is now have a *p*-axis component only, the average vector sum of the source imaginary power vector given in (23) is zero. This means that there is no phase difference between the fundamental positive sequence of the source voltage and the load voltage, and thus the power looping in the UPQC system has a minimum value.

To investigate the amount of power taken by the parallel active filter and the power released from the series active filter, the waveforms of instantaneous active power of $p_{shunt}$, $p_{series}$, $p_S$, $p_l$ and the waveform of output of integral controller are obtained.

In Fig. 19(b), the average value of instantaneous active power of the load is constant after the transient process when that UPQC starts to operate at 0.1s. It is because the load voltage is compensated to be balanced and sinusoidal, thus there is no influence to the load side even if there is a fault at the source side. When the fault occurs at 0.2s, the

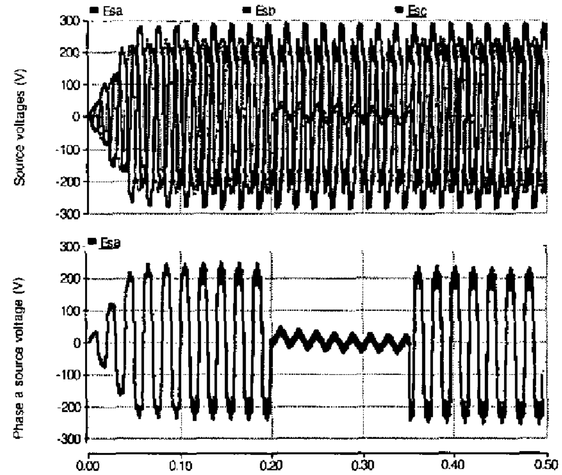


Fig. 14: Waveforms of source voltages. A fault occurs at 0.2s in phase a and is cleared at 0.35s

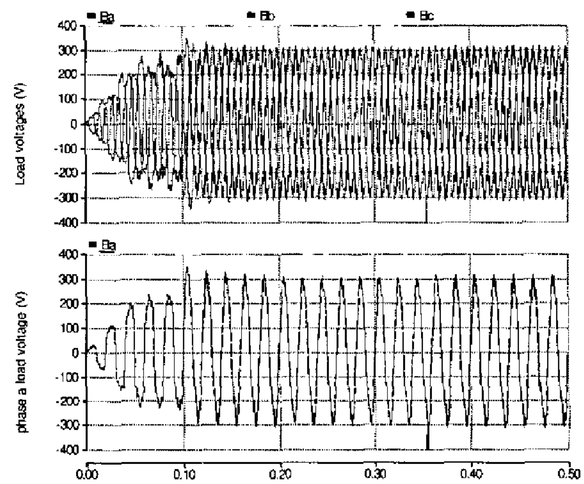


Fig. 15: Waveforms of load voltages

series active filter needs more active power from the dc link as in Fig. 21, and thus there is an increase in the power drawing from the dc storage. The integration controller compensates this power with a transient process as in Fig. 22, and the drop of dc stops after the system reaches its steady state as in Fig. 23(a). As expected, the steady-state dc voltage will be lower if the required power from the series active filter is larger. Notice that the dc component of $p_{shunt}$ in Fig. 20 is identical to the output of the integration controller in Fig. 22. The ac component of $p_{shunt}$ is used to compensate harmonics and imbalance of the load current. It is interesting to see that in Fig. 19 the average active power supplied by the source is approximately the same before and after the fault in steady-state, except that there may be a difference in the loss power. This shows that the power looping in the UPQC does not consume extra power from the source, however, as shown in Fig. 18, the increase in $p_{series}$ will increase the magnitude of source current, since the total amount of active power flowing through the system $p_{upqc}$ is larger.

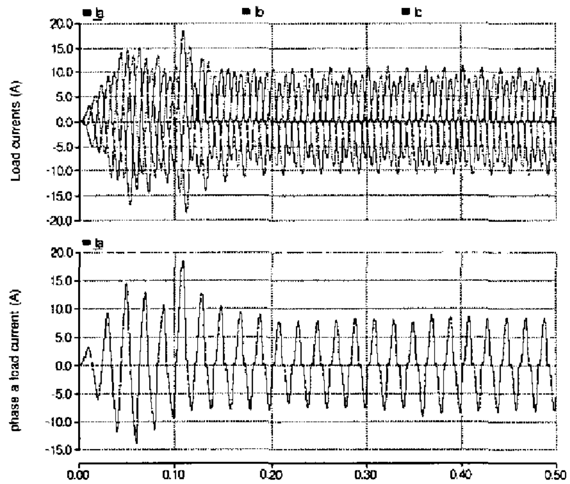

Fig. 16: Waveforms of load currents

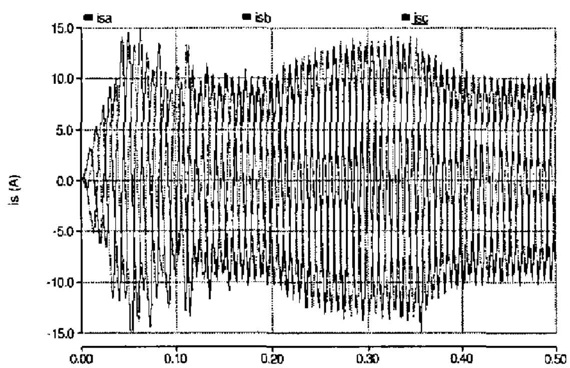

Fig. 17: Waveforms of source current

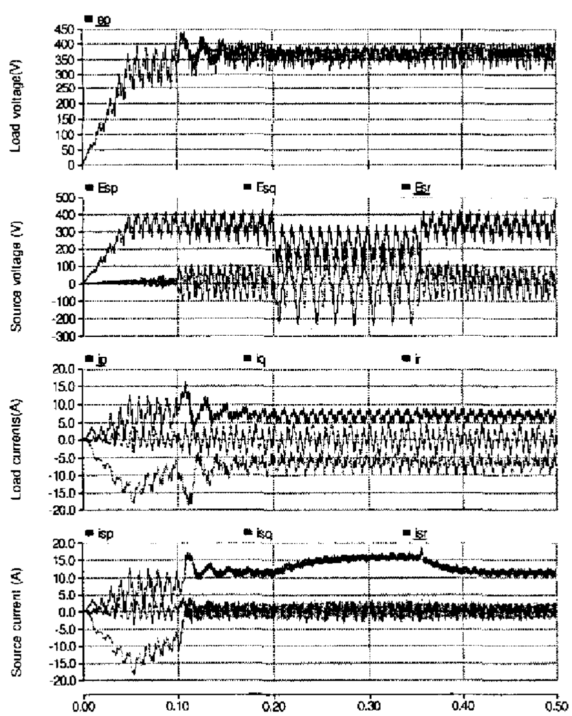

Fig. 18: Waveforms of voltages and currents in $p$-$q$-$r$ reference frames

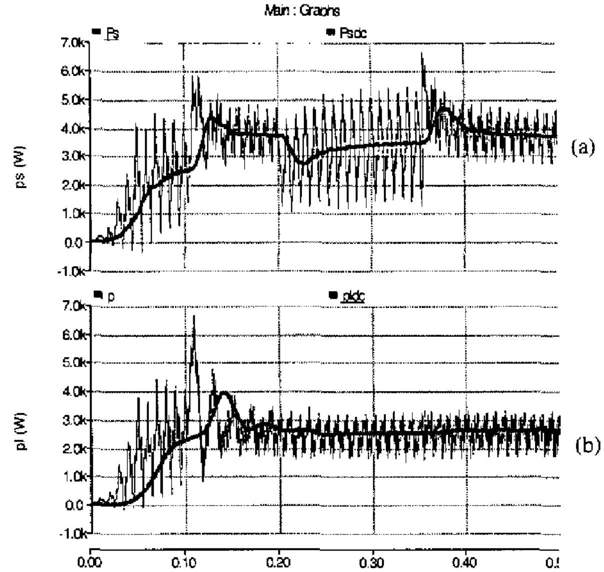

Fig. 19: Waveforms of (a) source power and (b) load power

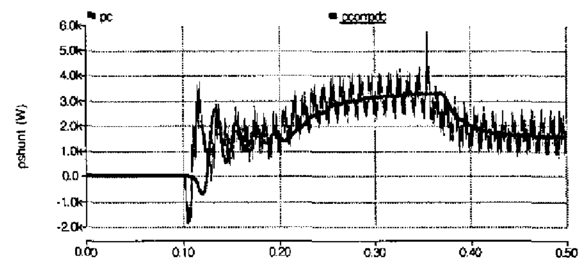

Fig. 20: Waveform of parallel active filter power $p_{shunt}$

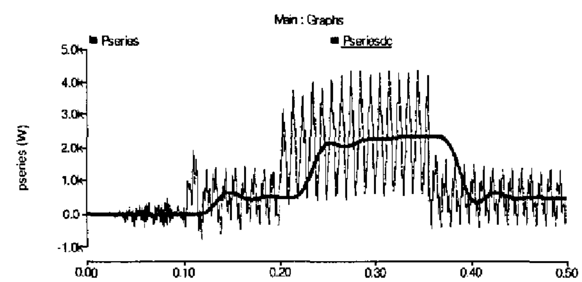

Fig. 21: Waveform of series active filter power $p_{series}$

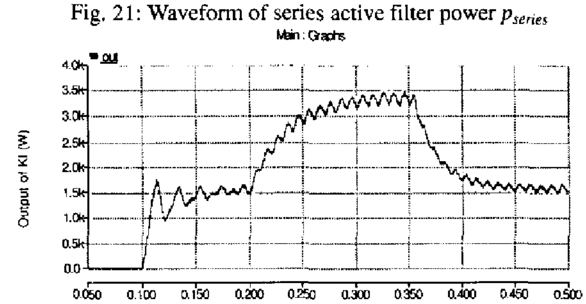

Fig. 22: Waveform of integral controller output

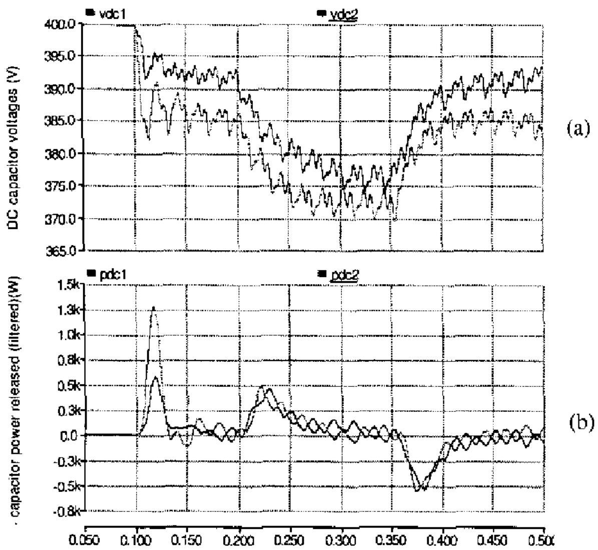


Fig. 23: Waveforms of (a) dc capacitor voltages, (b) filtered dc capacitor power released

## CONCLUSION

The instantaneous active power flow in a UPQC without rectifier support in three-phase four-wire system is analyzed by *p-q-r* theory. The voltage detection and modified current detection methods are combined to maintain a zero average dc storage power. A proposed *q*-axis component is added to place the current vector to align with the balanced sinusoidal reference. A negative integration feedback control of dc storage power is proposed, and its equivalent model by dc voltage feedback is analyzed. The above control algorithms are simulated and the results agreed well with the theoretical expectation. The future work includes the analysis of the effect of sampling and quantization error on detection of dc storage voltage, minimization of looping active power and loss power in the UPQC system, and the consideration of power flow at the source side.

## REFERENCES


[1] D. Sabin and A. Sundaram, "Quality enhances reliability", *IEEE Spectrum*, vol. 33, no. 2, pp. 34-41, Feb. 1996.

[2] H. Akagi, "New trends in active filters for power conditioning", IEEE Trans. Ind. Applicat., vol. 32, no. 6, pp. 1312-1322, Nov./Dec. 1996.

[3] H. Akagi, Y. Kanazawa, and A. Nabae, "Instantaneous reactive power compensators comprising switching devices without energy storage components", IEEE Trans. Ind. Aplicat., vol. 20, pp. 625-630, May/June 1984.

[4] F. Z. Peng and J. S. Lai, "Generalized instantaneous reactive power theory for three-phase power systems," IEEE Trans. Instrum. Meas., vol. 45, pp. 293–297, Feb. 1996.

[5] F. Z. Peng, G.W. Ott Jr, and D. J. Adams, "Harmonic and reactive power compensation based on the generalized instantaneous reactive theory for three-phase four-wire systems," IEEE Trans. Power Electron., vol. 13, pp. 1174–1181, Nov. 1998.

[6] Man-Chung Wong, Ying-Duo Han and Liang-Bing Zhao, "Study of distribution system unified conditioner (DS-UniCon)", Power Electronics and Motion Control Conference, 2000. Proceedings. PIEMC 2000. The Third International, Volume: 3 , 15-18 Aug. 2000

[7] Man-Chung Wong, Chang-Jiang Zhan, Ying-Duo Han and Liang-Bing Zhao, "A unified approach for distribution system conditioning: Distribution System Unified Conditioner (DS-UniCon)", Power Engineering Society Winter Meeting, 2000. IEEE, Volume: 4 , 23-27 Jan. 2000

[8] Hideaki Fujita and Hirofumi Akagi, "The unified power quality conditioner: The integration of series- and shunt-active filters", IEEE Transactions on power Electronics, vol. 13, no. 2, Mar. 1998

[9] Arindam Ghosh and Gerard Ledwich, "Compensation of Distribution System Voltage Using DVR", IEEE transactions on power delivery, vol. 17, no. 4, Oct. 2002

[10] H. S. Kim and H. Akagi, "The instantaneous power theory on the rotating p-q-r reference frames", in proc. IEEE/PEDS'99 Conf., Hong Kong, July 1999, pp. 422-427

[11] H. Kim, F. Blaabjerg, and B. Bak-Jensen, "Spectral Analysis of instantaneous powers in single-phase and three-phase systems with use of p-q-r theory", IEEE Trans. Power Electron., vol. 17, pp. 711-720, Nov. 2002

[12] H. Kim, F. Blaabjerg, B. Bak-Jensen and Jaeho Choi, "Instantaneous power compensation in three-phase systems by using p-q-r theory", IEEE Transactions on Power Electronics, vol. 17, no. 5, Sep. 2002

[13] Hyosung Kim, Sang-Joon Lee and Seung-Ki Sul, "A calculation for the compensation voltages in dynamic voltage restorers by use of PQR power theory", Applied Power Electronics Conference and Exposition, 2004. APEC '04. Nineteenth Annual IEEE , Volume: 1 , 22-26 Feb. 2004

[14] Sang-Joon Lee, Hyosung Kim, Seung-Ki Sul and Blaabjerg, F.; "A novel control algorithm for static series compensators by use of PQR instantaneous power theory", IEEE Transactions on Power Electronics, Volume: 19, Issue: 3 , May 2004

[15] Chi-Seng Lam, Man-Chung Wong, Ying-Duo Han, "Dynamic Voltage Restorer (DVR)", Symposium on Applied Science and Technology Development in Macau 2004