Fundamental Study of 3 Dimensional Pulse Width Modulation

Man-Chung Wong Jing Tang Ning-Yi Dai Ying-Duo Han

***Abstract*—In this paper, fundamental study of 3 Dimensional Pulse Width Modulation (3DPWM) is achieved. It depends on the dc linked capacitor center-split structure inverter. The mathematical model of 3DPWM for 2-level system in rectangular, cylindrical and spherical coordinates is studied. The equivalent circuits in α,β and 0 frames are expressed independently. The conventional 2DPWM and this novel 3DPWM are compared. Generalized 3DPWM in Multi-level system is investigated. The 3D Space Vector Modulation is considered with simulation results too. This paper forms the basic background for the further study of 3DPWM. The advantages and the disadvantages of this novel 3DPWM are also included.**

***Index Terms*—Active Filter, Multi-level System, Power Quality Compensator, Pulse Width Modulation, Three-Phase Converter, Three-Phase Four-Wire System**

I. INTRODUCTION

IN the past researches of the Power Electronics Control techniques, almost all of them are focused on 3-Phase 3-Wired System. It means that there are a lot of researches focusing on the 2-Dimensional Pulse Width Modulation Techniques [1][2]. However, conventional control techniques of 2DPWM cannot handle the issues in 3-Phase 4-Wired systems. In order to overcome the 3-phase 4-wired system issues, there are two ways to handle the current through the returned ground or neutral wire. One is the 4-Armed Converter and the other is the Split DC Link structure. In most recent researches, 4-Armed Converters are proposed to handle 3-Phase 4-Wired issues including one dedicated arm for neutral line current compensation [3][4][5]. In this paper, 3-Armed Converter is focused with 3DPWM technique [6] to overcome the 3-Phase 4-Wired System issues with the DC Center-Split structure. The fundamental study of 3DPWM is performed, including the circuit power flow, control algorithm and implementation so as to fabricate the basic theory of 3 Dimensional Pulse Width Modulation techniques in DC Link Center-Split Inverter.

Section II will describe the mathematical model of 3DPWM in rectangular, cylindrical and spherical coordinates, and power flow in dc link center-split inverter is also illustrated. The conventional 2DPWM and this novel 3DPWM are compared in Section III. Section IV shows the generalized 3DPWM theory in multi-level systems. The novel 3 Dimensional Space Vector Modulation (3DSVM) control is achieved. Simulation results are shown in Section VI.

II. MATHEMATICAL MODEL OF 3DPWM AND POWER FLOW IN DC LINK CENTER-SPLIT INVERTER

In this paper, the DC Link Center-Split Inverter structure will be the basic configuration for the discussion of 3 Dimensional Pulse Width Modulation (3DPWM). Fig. 1 shows the DC Link Center-Split Inverter in 2-level as a shunted connected Power Quality Compensator in 3-phase 4-wire system to compensate the current issues such as the unbalance, reactive, harmonics and neutral currents.

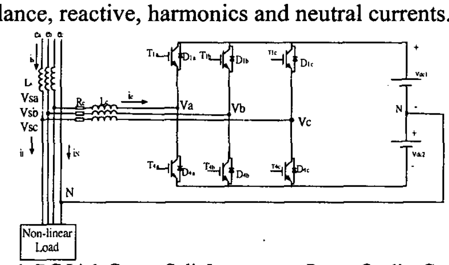

Fig. 1 DC Link Center-Split Inverter as a Power Quality Compensator

A. Mathematical Model of 3DPWM

The instantaneous voltage in α-β-0 can be transformed from a-b-c frame by matrix [P], such as shown in (1)

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = [P] \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \qquad (1)$$

where

$$[P] = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

When the instantaneous equivalent voltage vector $\vec{v}$ is considered in α-β-0 frame, it can be expressed as (2).

$$\vec{v} = v_\alpha \vec{n}_\alpha + v_\beta \vec{n}_\beta + v_0 \vec{n}_0 \qquad (2)$$

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The $\{\vec{n}_\alpha, \vec{n}_\beta, \vec{n}_0\}$ forms a basis and they are orthogonal to each other such that $\vec{n}_0 \bullet \vec{n}_\alpha = \vec{n}_0 \bullet \vec{n}_\beta = \vec{n}_\alpha \bullet \vec{n}_\beta = 0$. In 3-Dimensional aspect, the instantaneous equivalent voltage vector $\vec{v}$ can be expressed in Rectangular, Cylindrical or Spherical Coordinates respectively according to the basis $\{\vec{n}_\alpha, \vec{n}_\beta, \vec{n}_0\}$. Referring to the Fig.1, it is assumed that the DC-Linked upper-arm and lower-arm voltage of the Inverter are equal to each other, such as $V_{dc1} = V_{dc2} = V_{dc}$.

Due to the concept of the switching function and a-b-c frame transformation, the instantaneous equivalent voltage vector can be given as (3).

$$\vec{v}=V_{dc}\left[\sqrt{\frac{2}{3}}\left(S_a-\frac{1}{2}S_b-\frac{1}{2}S_c\right)\cdot\vec{n}_\alpha+\frac{1}{\sqrt{2}}(S_b-S_c)\cdot\vec{n}_\beta+\frac{1}{\sqrt{3}}(S_a+S_b+S_c)\cdot\vec{n}_0\right] \quad (3)$$

where $S_x \in \{1,-1\}$ in 2-level system and x can be a, b and c.

B. Rectangular Coordinate

There are totally 8 voltage vectors in 2-level Inverter. In Rectangular Coordinate, the $\{\vec{n}_\alpha, \vec{n}_\beta, \vec{n}_0\}$ forms the basis and is shown in Fig. 2. Fig. 3 shows the voltage vector's allocation in 3-dimensional aspect.

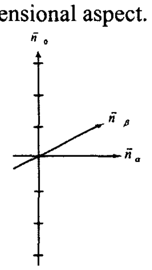

Fig. 2 Basis $\{\vec{n}_\alpha \cdot \vec{n}_\beta \cdot \vec{n}_0\}$

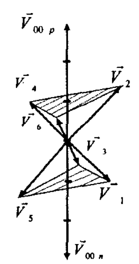

Fig. 3 3DPWM Voltage Vectors

	S_a	S_b	S_c	S_α	S_β	S_0
$\vec{V}_1$	1	-1	-1	2	0	-1
$\vec{V}_2$	1	1	-1	1	2	1
$\vec{V}_3$	-1	1	-1	-1	2	-1
$\vec{V}_4$	-1	1	1	-2	0	1
$\vec{V}_5$	-1	-1	1	-1	-2	-1
$\vec{V}_6$	1	-1	1	1	-2	1
$\vec{V}_{0p}$	1	1	1	0	0	3
$\vec{V}_{00n}$	-1	-1	-1	0	0	-3

TABLE I

2-LEVEL 3DPWM VOLTAGE VECTORS IN RECTANGULAR COORDINATE

Refer to Fig. 3, the values of S_α, S_β and S_0 can be expressed as equations (4), (5) and (6) respectively.

$$S_\alpha = S_a - \frac{1}{2}S_b - \frac{1}{2}S_c \quad (4)$$

$$S_\beta = S_b - S_c \quad (5)$$

$$S_0 = S_a + S_b + S_c \quad (6)$$

C. Cylindrical Coordinate

The voltage vector's allocation in 3-dimensional aspect can be expressed in Cylindrical Coordinate. Furthermore, equation (3) can be described as (7).

$$\vec{v} = Z_\alpha \vec{n}_\alpha + Z_\beta \vec{n}_\beta + Z_0 \vec{n}_0 \quad (7)$$

,where

$$Z_\alpha = V_{dc}\cdot\sqrt{\frac{2}{3}}S_\alpha \qquad Z_\beta = V_{dc}\cdot\frac{1}{\sqrt{2}}S_\beta \qquad Z_0 = V_{dc}\cdot\frac{1}{\sqrt{3}}S_0$$

In Cylindrical Coordinate, r, θ and Z_0 are the parameters to describe the vector's allocation in space. In Tables 2 and 3, V_{dc} is assumed to be 1 and the symbol " * " means "undefined".

$$r = \sqrt{Z_\alpha^2 + Z_\beta^2} \quad (8)$$

$$\theta = \tan^{-1}\left(\frac{Z_\beta}{Z_\alpha}\right) \quad (9)$$

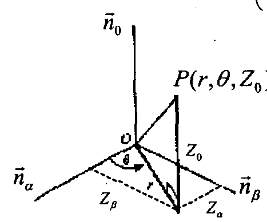

Fig. 4 Cylindrical Coordinate

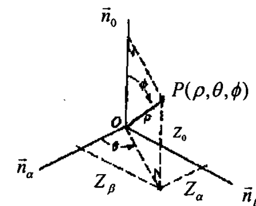

Fig. 5 Spherical Coordinate

V_{dc}=1	r	θ	Z_0
$\vec{V}_1$	1.633	0^0	-0.577
$\vec{V}_2$	1.633	60^0	0.577
$\vec{V}_3$	1.633	120^0	-0.577
$\vec{V}_4$	1.633	180^0	0.577
$\vec{V}_5$	1.633	240^0	-0.577
$\vec{V}_6$	1.633	300^0	0.577
$\vec{V}_{00p}$	0	*	$\sqrt{3}$
$\vec{V}_{00n}$	0	*	$-\sqrt{3}$

TABLE 2

V_{dc}=1	ρ	θ	ϕ
$\vec{V}_1$	$\sqrt{3}$	0^0	109.5^0
$\vec{V}_2$	$\sqrt{3}$	60^0	70.5^0
$\vec{V}_3$	$\sqrt{3}$	120^0	109.5^0
$\vec{V}_4$	$\sqrt{3}$	180^0	70.5^0
$\vec{V}_5$	$\sqrt{3}$	240^0	109.5^0
$\vec{V}_6$	$\sqrt{3}$	300^0	70.5^0
$\vec{V}_{00p}$	$\sqrt{3}$	*	0^0
$\vec{V}_{00n}$	$\sqrt{3}$	*	180^0

TABLE 3

D. Spherical Coordinate

The voltage vector's allocation in 3-dimensional aspect can be expressed in Spherical Coordinate. The ρ, θ and ϕ are the parameters to describe the vector's allocation in Spherical Coordinate space, shown in Fig. 5.

$$\rho = \sqrt{Z_\alpha^2 + Z_\beta^2 + Z_0^2} \quad (10)$$

$$\phi = \cos^{-1}\left(\frac{Z_0}{\rho}\right) = \sin^{-1}\left(\frac{r}{\rho}\right) \quad (11)$$

E. Equivalent Circuits in α–β–0 Frames

Fig. 6 shows the Equivalent Circuits in α–β–0 frames respectively. There are 5 levels {2,1,0,-1,-2}, 3 levels {2,0,-2} and 4 levels {3,1,-1,-3} in α, β and 0 equivalent circuits respectively. The different voltage levels will generate the different current amplitudes in different equivalent circuits. However, when the compensated currents in different circuits are all positive, the vector $\vec{V}_2$ can be chosen. But, in another case, the required compensated currents are positive in α, positive in β and negative in 0. No simple unique vector can be chosen. The compensated vector may not be uniquely defined and it may enhance one improvement in one direction but it intends a dedicated error in another direction.

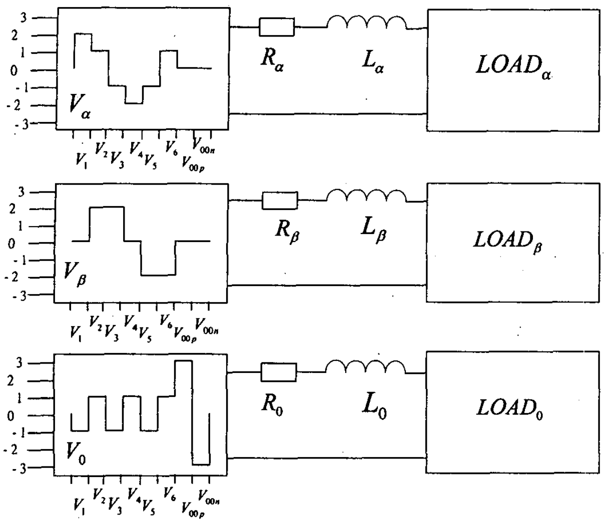

Fig. 6 Equivalent Circuits in α–β–0 frames

F. Power Flow in DC Link Center-Split Inverter and 3DPWM

The DC Link Capacitor voltage is considered in this section. Fig. 7.a shows a simplified circuit for upper-arm capacitor voltage when the upper-arm switching component is turned on. Fig. 7.b shows another case when the lower-arm capacitor voltage is discharged.

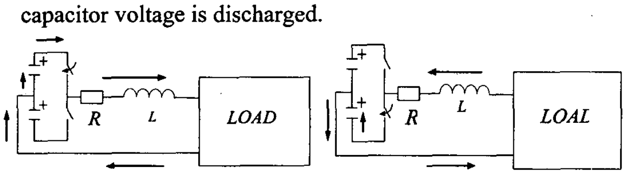

Fig. 7.a Fig.7.b

When the product of upper-arm discharged current and switching time equals to the product of lower-arm discharged current and switching time, the upper-arm and lower-arm voltages will be balanced. The neutral line current may be the key control parameter to make the upper-arm and lower-arm voltages to be balanced. There are 2 cases: 1) The Center-Split Inverter to be an independent voltage source for 3-phase 4-wired system, 2) it acts as a power quality compensator. For the first case, when the neutral line current is zero in average so that the upper-arm and lower-arm voltages will be balanced automatically. It reaches the normal operation requirement in the 3-Phase 4-wire system; the neutral line current should be zero. For the second case, it acts as the power quality compensator that injected the reversed equal-amplitude current into the coupling point to ensure that the sinusoidal current existed between the voltage source and the coupling point. However, the neutral line current in the inverter cannot be null, as it needs to inject the current into the source neutral wire so that the upper-arm and lower-arm voltages cannot be balanced in the ideal case.

III. Conventional 2DPWM and Novel 3DPWM in 2-Level System

In Conventional PWM, the 2-level voltage space vectors are shown in Fig. 8. There are totally 8 vectors, but 2 are zero vectors. The number of states in 2-level system is 7 and totally there are 8 available vectors. The number of states is not equaled to the number of available vectors.

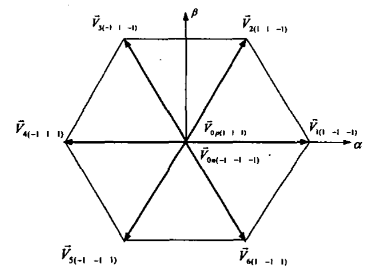

Fig. 8 2 Level Voltage Space Vectors

But, in 3DPWM, the number of 2-level voltage space vectors is equal to the number of states, shown in Fig. 3.

In consideration of the dc linked voltage amplitude, the conventional 2-level system will have doubled dc amplitude than center-split structure case. The modulation index of the center-split inverter is comparatively lower.

Under-modulation control in conventional 2-level case, the voltage reference is inside the hexagon boundary, shown in Fig. 8. But, in center-split structure system, the area bounded by the under-modulation is a 3 dimensional spatial region, shown in Fig. 9.

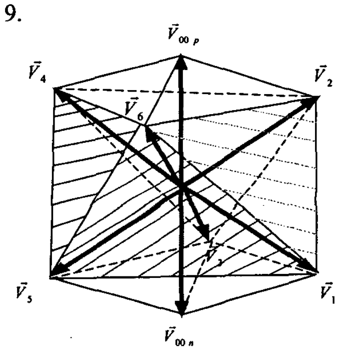

Fig. 9 Under-modulation region in 3DPWM

IV. Generalized 3DPWM in Multilevel System

In 2-level case, the switching function S_x can be given in {1,-1} for phase a, b and/or c respectively. The number of available states and vectors is 2^3 or 8. In 3-level system, S_x has {1,0,-1}. The number of vectors is 3^3 or 27. In conventional 2DPWM, the number of available states is 18. But in 3DPWM, the number of available states is equal to the number of vectors, 27. Fig. 9 shows the 3-level voltage space vectors in 3DPWM.

It is obvious that no matter which level system is under consideration; the number of available states will be equaled to the number of available vectors. In 5-level system, the number of available states and vectors is 5^3 or 125.

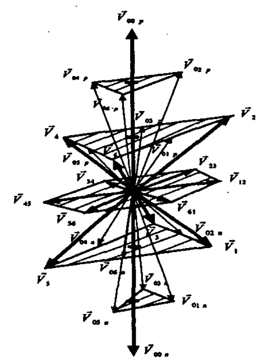

Fig. 10 3-level Voltage Space Vector's Allocation

Considering $\vec{n}_0$ axis in 2-level case, there are 4 altitudes: 2 planes and 2 points. In positive $\vec{n}_0$ basis, $\{\vec{V}_2, \vec{V}_4, \vec{V}_6\}$ forms a plane and $\vec{V}_{00p}$ settles a point in space. In negative way, $\{\vec{V}_1, \vec{V}_3, \vec{V}_5\}$ forms a plane and $\vec{V}_{00n}$ settles a point in space, shown in Fig. 3. By considering S_0 in TABLE 1, the altitudes are {3, 1, -1, -3}. Due to the same consideration in 3-level system, there are 7 altitudes, {3, 2, 1, 0, -1, -2, -3} in $\vec{n}_0$ axis. In 5 level, there are 13 altitudes, {6, 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5, -6} in $\vec{n}_0$ axis. It can be deduced that increasing each level will increase 3 more altitudes or planes in $\vec{n}_0$ axis for 3DPWM.

V. 3 Dimensional Space Vector Modulation Control Strategies

In 3D Space Vector Modulation (SVM) technique, the voltage-second reference can be estimated by a sequence of voltage-second states, shown in equations (12) and (13). In 3-D SVM, the vectors $\vec{V}_x$ and $\vec{V}_y$ can be chosen as the neighbor vectors defined as the conventional SVM.

$$\vec{V}^*_{ref} T_S = \vec{V}_x t_x + \vec{V}_y t_y + \vec{V}_0 t_0 + \vec{V}_{zero} t_{zero} \qquad (12)$$

$$t_{zero} = T_S - t_x - t_y - t_0 \qquad (13)$$

In (12), the $\vec{V}_{zero} t_{zero}$ is equal to zero in the sense that it does not take any action in the equivalent voltage-second reference and t_{zero} is equal to zero when the over-modulation PWM is considered. The reference vector, $\vec{V}^*_{ref}$, can be described in 3 dimensional aspect (14). Fig. 11 shows the space vector modulation concept in 3D.

$$\vec{V}^*_{ref} = \vec{V}^*_\alpha \vec{n}_\alpha + \vec{V}^*_\beta \vec{n}_\beta + \vec{V}^*_0 \vec{n}_0 \qquad (14)$$

Considering the defined angles and amplitudes in Fig. 11, the reference values could be determined according to the equation (15).

$$\begin{bmatrix} \vec{V}^*_\alpha \\ \vec{V}^*_\beta \\ \vec{V}^*_0 \end{bmatrix} = \begin{bmatrix} \vec{V}^*_{ref} \sin\phi\cos\theta \\ \vec{V}^*_{ref} \sin\phi\sin\theta \\ \vec{V}^*_{ref} \cos\theta \end{bmatrix} = \begin{bmatrix} \vec{V}^*_{\alpha\beta} \cos\theta \\ \vec{V}^*_{\alpha\beta} \sin\theta \\ \vec{V}^*_{ref} \cos\theta \end{bmatrix} \qquad (15)$$

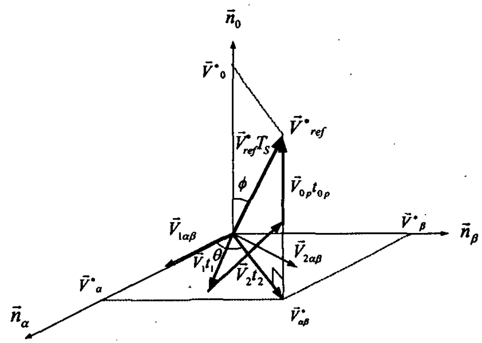

Fig. 11 Determination of Reference Voltage Vector

Due to the consideration that there is no exacted zero value vector in 2-level 3DPWM so that equation (12) should be express as (16).

$$\vec{V}^*_{ref} T_S = \vec{V}_x t_x + \vec{V}_y t_y + \vec{V}_{0n} t_{0n} + \vec{V}_{op} t_{op} \qquad (16)$$

There are two cases in equation (16). When negative neutral current is needed, one can receive $t_{0n} = t_0 + \frac{t_{zero}}{2}$ and $t_{0p} = \frac{t_{zero}}{2}$. On the other hand, $t_{0p} = t_0 + \frac{t_{zero}}{2}$ and $t_{0n} = \frac{t_{zero}}{2}$ are dedicated for positive neutral current. The consideration of equation (12) is taken as $\vec{V}_{zero} t_{zero}$ should be cancelled to each other in all axis-components.

Case 1: Under Modulation

The cycle time, Ts, is the summation (17) of t_x, t_y and t_{zero}. The t_{0n} and t_{0p} are calculated as (18) and (19) respectively in the cases of positive and negative zero vectors.

$$T_S = t_x + t_y + t_0 + t_{zero} \qquad (17)$$

$$\text{Required Positive Zero Vector: } \begin{cases} t_{0p} = t_0 + \frac{t_{zero}}{2} \\ t_{0n} = \frac{t_{zero}}{2} \end{cases} \qquad (18)$$

$$\text{Required Negative Zero Vector: } \begin{cases} t_{0p} = \frac{t_{zero}}{2} \\ t_{0n} = t_0 + \frac{t_{zero}}{2} \end{cases} \qquad (19)$$

Case 2: Overmodulation

In the case of overmodulation, the actual inverter switching times t'_x, t'_y, t'_0 and t_{zero} are simply computed from modified voltage vector using the geometrical relationship.

$$t'_x = \frac{t_x}{t_x + t_y + t_0} T_s \qquad (20)$$

$$t'_y = \frac{t_y}{t_x + t_y + t_0} T_s \qquad (21)$$

$$\text{Required Positive Zero Vector: } \begin{cases} t'_0 = t_{0p} = \frac{t_0}{t_x + t_y + t_0} T_s \\ t_{0n} = t_{zero} = 0 \end{cases} \qquad (22)$$

Required Negative Zero Vector: $$\begin{cases} t_0' = t_{0n} = \dfrac{t_0}{t_x + t_y + t_0} T_s \\ t_{0p} = t_{zero} = 0 \end{cases} \quad (23)$$

VI. Simulation Results

A simulation test is performed by 3-Dimensional Space Vector Modulation Technique as a Power Quality Compensator to compensate the harmonics, unbalance and reactive current as well as the neutral line current in 3-Phase 4-Wired System.

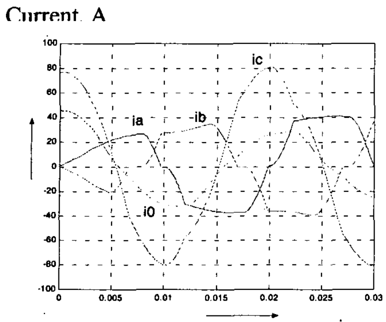

Fig. 12

Fig. 12 shows the nonlinear load current waveforms before the compensation.

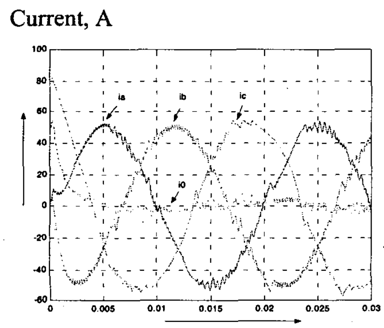

Fig. 13

Fig. 13 shows the waveforms after compensation by 5K Hz Switching 3 Dimensional Space Vector Modulation Control Strategy. It is obvious that this novel proposed control strategy in 3DSVM could reduce the harmonics, the reactive and unbalance current with eliminating neutral current as well.

VII. Conclusion

The fundamental study of 3DPWM is performed, including the circuit power flow, control algorithm and implementation so as to fabricate the basic theory of 3 Dimensional Pulse Width Modulation in DC Link Center-Split Inverter.

The mathematical model of 3DPWM is described in Rectangular, Cylindrical and Spherical Coordinates respectively. Conventional 2DPWM and this novel 3DPWM are compared. In novel 3DPWM, the number of states is equal to the number of available vectors. Generalized 3DPWM theory in multi-level systems is deduced.

The advantages by using DC Link Center-Split Inverter with 3DPWM than 4-leg Inverter are:

1) Fewer Switching Components are needed.
2) Comparatively, less complicated control strategy can be implemented due to only 2^3 or 8 space vectors that are available. In 4-legs system, there are totally 2^4 or 16 space vectors.
3) Less triggered pulse terminals are needed in the controller so that the controller's hardware requirement is lower.
4) Lower Cost.

The disadvantage of Center-Split Inverter is the dc linked upper-arm and lower-arm voltage balancing issue.

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Biographies

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