IEEE 1999 International Conference on Power Electronics and Drive Systems, PEDS'99, July 1999, Hong Kong.

Mathematical Model and Dual-DSP Control of Tri-Level PWM Reversible Rectifier

Zhan Changjiang *, Han Yu *, Xie Ting *, Zhao Zhengyi *, Wong Manchung **, Han Yinduo **

* Dept. of Electrical Engineering
Tsinghua University, Beijing
100084, P.R.China

** Dept. of Electrical and Electronic Engineering
University of Macao, Macao

Abstract - Based on the concept of switching function(SF) and space vector(SV), and cited the coordinate transformation matrix on space reference frame in the unified theory of an AC motor, a new mathematical model of the Tri_Level PWM Reversible Rectifier was presented in detail. The synchronous reference frame model was used to achieve the *dq* decoupling control of the converter. All control algorithms were implemented in dual digital-signal processors (DSP). The validity of the system model was verified by the results of computer-aided simulation and experiment.

I. INTRODUCTION

Compared with conventional two-level PWM reversible rectifier, the tri-level converter is able to reduce harmonic currents without requiring the power devices such as GTO or IGBT to be operated at a high switching frequency, and is very suitable for high voltage systems because of its Neutral-Point-Clamped (NPC) main circuit structure. Thus, it plays a greater role in high power and high voltage applications where the switching frequency is limited around or below several hundreds hertz [1]-[4].

Although lots of correlative publications have been presented, the study on tri-level converter just comes up to the surface. The common PWM control strategies include harmonic elimination [4], optimal [3], space vector PWM [2] and hysteresis current control [1]. Harmonic elimination method is very simple but its dynamic response is too slow. Optimal PWM method is suitable for high index modulation but it is not fit for middle modulation because of many jumps and large low frequency harmonics. Hysteresis current control based on voltage space vector has features such as fast response and easy implementation. However, it still remains the main drawbacks of hysteresis control.

In this paper, based on the concept of switching function (SF) and space vector (SV), and cited the coordinate transformation matrix on space reference frame in the unified theory of an AC motor, a new mathematical model of the Tri_Level PWM reversible rectifier was presented in detail. The synchronous reference frame model was used to achieve the *dq* decoupling control of the converter. All control algorithms were implemented in dual digital-signal processors (DSP). The validity of the system model was verified by the results of computer-aided simulation and experiment.

II. MATHEMATICAL MODEL OF TRI-LEVEL CONVERTER

A. Model in the abc Frame

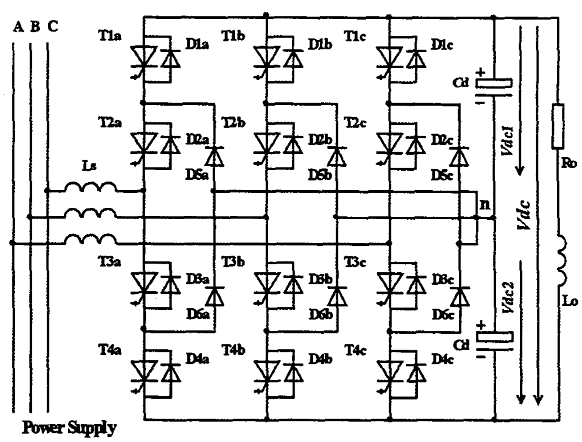

Fig. 1 A Tri-level PWM Reversible Rectifier

Fig. 1 shows a Tri-level reversible rectifier which consists of RL DC load. The three phase utility voltage is assumed to be balanced and symmetrical. Then $V_{SA} = V_{Sm} * \cos(\omega t)$, ω is

0-7803-5769-8/99/$10.00©1999IEEE

the source positive sequence fundamental voltage angular frequency. The first A phase switching function Sa of power devices $T_{1a}\sim T_{4a}$ is defined as [5]:

$$S_a = \begin{cases} 1, & T_{1a}、T_{2a}\ \text{ON and}\ T_{3a}、T_{4a}\ \text{OFF}; \\ 0, & T_{2a}、T_{3a}\ \text{ON and}\ T_{1a}、T_{4a}\ \text{OFF}; \\ -1, & T_{3a}、T_{4a}\ \text{ON and}\ T_{1a}、T_{2a}\ \text{OFF}; \end{cases} \tag{1}$$

In order to obtain the mathematical model of system, Sa may be written as:

(1) . if $S_a = 1$, then $S_{1a} = 1$, $S_{2a} = 0$, $S_{3a} = 0$;
(2) . if $S_a = 0$, then $S_{1a} = 0$, $S_{2a} = 0$, $S_{3a} = 1$;
(3) . if $S_a = -1$, then $S_{1a} = 0$, $S_{2a} = 1$, $S_{3a} = 0$;

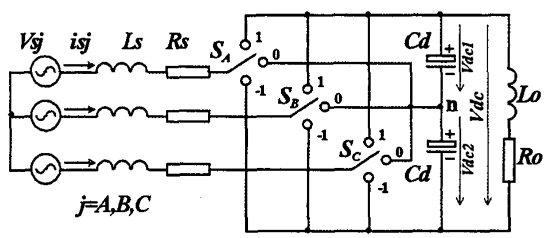

Fig. 2 The Equivalent Circuit of Tri-level Rectifier

In the Fig.2, the boundary condition of S_{1a}, S_{2a} and S_{3a} is defined as:

$$\begin{cases} S_{1a} + S_{2a} + S_{3a} = 1 \\ S_{1a} = 1\, or\, 0, S_{2a} = 1\, or\, 0, S_{3a} = 1\, or\, 0 \end{cases} \tag{2}$$

The balanced three phase system has no neutral connection, then it will be

$$\begin{cases} V_{sA} + V_{sB} + V_{sC} = 0 \\ i_{sA} + i_{sB} + i_{sC} = 0 \end{cases} \tag{3}$$

and $$V_{N0} = -\left(V_{RN} + V_{SN} + V_{TN}\right)/3 \tag{4}$$

So a general mathematical model of the tri-level PWM reversible rectifier is established as follows:

$$Z\dot{X} = A \cdot X + B \cdot e \tag{5}$$

where

$$B = diag\begin{bmatrix}1 & 1 & 1 & 0 & 0 & 0\end{bmatrix}$$
$$e = \begin{bmatrix}V_{SA} & V_{SB} & V_{SC}\end{bmatrix}^T$$

$$Z = diag\left[L_s\ L_s\ L_s\ C_d\ C_d\ L_0\right]$$
$$X = \left[i_{SA}\ i_{SB}\ i_{SC}\ V_{dc1}\ V_{dc2}\ I_0\right]^T$$

$$A = \begin{bmatrix} -R_s & 0 & 0 & -d_{11} & d_{12} & 0 \\ 0 & -R_s & 0 & -d_{21} & d_{22} & 0 \\ 0 & 0 & -R_s & -d_{31} & d_{32} & 0 \\ S_{1a} & S_{1b} & S_{1c} & 0 & 0 & -1 \\ -S_{2a} & -S_{2b} & -S_{2c} & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & -R_0 \end{bmatrix}$$

$$d_{11} = (S_{1a} - f_1); \quad d_{21} = (S_{1b} - f_1); \quad d_{31} = (S_{1c} - f_1);$$

$$d_{12} = (S_{2a} - f_2); \quad d_{21} = (S_{2b} - f_2); \quad d_{32} = (S_{2c} - f_2);$$

$$f_1 = \frac{S_{1a} + S_{1b} + S_{1c}}{3}; \qquad f_2 = \frac{S_{2a} + S_{2b} + S_{2c}}{3}.$$

B. Model in the Stationary $\alpha\beta$ Frame

In studying AC machine, certain quantities are found to have variables which are sinusoidal in the time and are often distributed in space. For three phase reversible rectifier, all variables are defined in the time but not in space. However, it is still beneficial to quote the concept of space vector for studying rectifiers or converters. Cited the coordinate transformation matrix on space reference frame in the unified theory of an AC motor, the "space " can be mathematically regarded as a complex plane rather than a "physical space". Define Park space vector as follows:

$$\begin{cases} \vec{V}_S = \sqrt{\dfrac{2}{3}}\left(V_{SA} + \alpha \cdot V_{SB} + \alpha^2 \cdot V_{SC}\right) \\ \vec{i}_S = \sqrt{\dfrac{2}{3}}\left(i_{SA} + \alpha \cdot i_{SB} + \alpha^2 \cdot i_{SC}\right) \end{cases} \tag{6}$$

where $\alpha = e^{j\frac{2\pi}{3}}$, $\alpha^2 = e^{-j\frac{2\pi}{3}}$,

Then, $$\begin{cases} \vec{V}_S = L_s \dfrac{d\vec{i}_s}{dt} + R_s \vec{i}_s + \vec{V}_R \\ \vec{V}_R = \vec{V}_{R1} - \vec{V}_{R2} \end{cases} \tag{7}$$

and $$\begin{cases} \vec{V}_{R1} = V_{dc1} \cdot (S_{1\alpha} + jS_{1\beta}) \\ \vec{V}_{R2} = V_{dc2} \cdot (S_{2\alpha} + jS_{2\beta}) \end{cases} \tag{8}$$

With

$$\begin{cases} S_{1\alpha}+jS_{1\beta}=\sqrt{\dfrac{2}{3}}\left[\left(S_{1a}-\dfrac{1}{2}S_{1b}-\dfrac{1}{2}S_{1c}\right)+j\left(\dfrac{\sqrt{3}}{2}S_{1b}-\dfrac{\sqrt{3}}{2}S_{1c}\right)\right] \\ S_{2\alpha}+jS_{2\beta}=\sqrt{\dfrac{2}{3}}\left[\left(S_{2a}-\dfrac{1}{2}S_{2b}-\dfrac{1}{2}S_{2c}\right)+j\left(\dfrac{\sqrt{3}}{2}S_{2b}-\dfrac{\sqrt{3}}{2}S_{2c}\right)\right] \end{cases}$$

and

$$\begin{cases} C_d\dfrac{dV_{dc1}}{dt}=S_{1\alpha}\cdot i_{s\alpha}+S_{1\beta}\cdot i_{s\beta}-I_0 \\ C_d\dfrac{dV_{dc2}}{dt}=-(S_{2\alpha}\cdot i_{s\alpha}+S_{2\beta}\cdot i_{s\beta})-I_0 \end{cases} \tag{9}$$

So a mathematical model of the tri-level system in the stationary $\alpha\beta$ frame is established as follows:

$$Z\dot{X}=AX+Be \tag{10}$$

where

$$Z=diag\begin{bmatrix}L_s & L_s & C_d & C_d & L_0\end{bmatrix}$$
$$X=\begin{bmatrix}i_{s\alpha} & i_{s\beta} & V_{dc1} & V_{dc2} & I_0\end{bmatrix}^T$$
$$B=diag\begin{bmatrix}1 & 1 & 0 & 0 & 0\end{bmatrix}$$
$$e=\begin{bmatrix}V_{s\alpha} & V_{s\beta}\end{bmatrix}^T$$

$$A=\begin{bmatrix} -R_s & 0 & -S_{1\alpha} & S_{2\alpha} & 0 \\ 0 & -R_s & -S_{1\beta} & S_{2\beta} & 0 \\ S_{1\alpha} & S_{1\beta} & 0 & 0 & -1 \\ -S_{2\alpha} & -S_{2\beta} & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & -R_0 \end{bmatrix}$$

C. *Model in the Rotating dq Frame*

The transformation relation can be given as followings:

$$T_{dq/\alpha\beta}=T_{\alpha\beta/dq}^{-1}=\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix} \tag{11}$$

$$\begin{cases} L_s pi_{sd}=-R_s i_{sd}+\omega L_s\cdot i_{sq}-V_{Rd}+V_{sd} \\ L_s pi_{sd}=-R_s i_{sq}-\omega L_s\cdot i_{sd}-V_{Rq}+V_{sq} \end{cases} \tag{12}$$

where $p=d/dt$,

$$\begin{cases} V_{Rd}=V_{Rd1}-V_{Rd2} \\ V_{Rq}=V_{Rq1}-V_{Rq2} \end{cases} \tag{13}$$

and

$$\begin{cases} \begin{bmatrix}S_{d1} & S_{q1}\end{bmatrix}^T=T_{dq/\alpha\beta}\cdot\begin{bmatrix}S_{\alpha1} & S_{\beta1}\end{bmatrix}^T \\ \begin{bmatrix}S_{d2} & S_{q2}\end{bmatrix}^T=T_{dq/\alpha\beta}\cdot\begin{bmatrix}S_{\alpha2} & S_{\beta2}\end{bmatrix}^T \end{cases} \tag{14}$$

Then it will be

$$\begin{cases} L_s\dfrac{di_{sd}}{dt}=-R_S i_{sd}+\omega L_s i_{sq}-V_{dc1}\cdot S_{d1}+V_{dc2}\cdot S_{d2}+V_{sd} \\ L_s\dfrac{di_{sq}}{dt}=-R_S i_{sq}-\omega L_s i_{sd}-V_{qc1}\cdot S_{q1}+V_{dc2}\cdot S_{q2} \end{cases}$$

with

$$\begin{cases} C_d\dfrac{dV_{dc1}}{dt}=S_{d1}\cdot i_{sd}+S_{q1}\cdot i_{sq}-I_0 \\ C_d\dfrac{dV_{dc2}}{dt}=-(S_{d2}\cdot i_{sd}+S_{q2}\cdot i_{sq})-I_0 \end{cases} \tag{15}$$

So a mathematical model of the tri-level system in the rotating dq frame is established as follows:

$$Z\dot{X}=AX+Be \tag{16}$$

where

$$Z=diag\begin{bmatrix}L_s & L_s & C_d & C_d & L_0\end{bmatrix}$$
$$X=\begin{bmatrix}i_{sd} & i_{sq} & V_{dc1} & V_{dc2} & I_0\end{bmatrix}^T$$

$$A=\begin{bmatrix} -R_s & \omega L_s & -S_{d1} & S_{d2} & 0 \\ -\omega L_s & -R_s & -S_{q1} & S_{q2} & 0 \\ S_{d1} & S_{q1} & 0 & 0 & -1 \\ -S_{d2} & -S_{q2} & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & -R_0 \end{bmatrix}$$

$$B=diag\begin{bmatrix}1 & 1 & 0 & 0 & 0\end{bmatrix}$$
$$e=\begin{bmatrix}\sqrt{\dfrac{3}{2}}V_{sm} & 0\end{bmatrix}^T$$

$$\begin{cases} V_{sd}=\sqrt{\dfrac{3}{2}}\cdot V_{Sm} \\ V_{sq}=0 \end{cases}$$

with the steady state equations: $\begin{cases} V_{Rd}=V_{sd}-R_s\cdot i_{sd} \\ V_{Rq}=-\omega L_s i_{sd} \end{cases}$

and $i_{sq}=0$ (17)

It should be pointed out that the power supply here is ideal.

III. SYNCHRONOUS *dq* DECOUPLING CONTROL

By contrast to other current controller, the rotating frame controller eliminates the steady- state error since it operates on dc quantities. Furthermore, the main idea of the *dq* decoupling control is: the cross coupling between the *d* and *q* axes due to inductance *Ls* was compensated by using feed-forward signals, then the control of *d* and *q* axes current depended on its own controller such as PI controller. The decoupling model of the tri-level PWM reversible rectifier is defined as follows:

$$\begin{cases} u_1 = V_{sd} - V_{Rd} + \omega\, L_s\, i_{sq} \\ u_2 = V_{sq} - V_{Rq} - \omega\, L_s\, i_{sd} \end{cases} \tag{18}$$

$$\begin{cases} L_s\, p\, i_{sd} = -R_s * i_{sd} + u_1 \\ L_s\, p\, i_{sq} = -R_s * i_{sq} + u_2 \end{cases} \tag{19}$$

A functional control diagram of the tri-level reversible rectifier is presented in Fig. 3. With the instantaneous reactive theory proposed by H. Akagi, the instantaneous reactive and active power *pq* control can be realized by the control of i_{sd} and i_{sq}. By means of the decoupling control of i_{sd} and i_{sq}, the tri-level reversible rectifier can achieve active and reactive or imaginary power control independently. Using conventional PI parameters, two decoupled current loop controller can be easily developed. The outer DC voltage close loop control maintains the DC bus voltage with compensating the circuit power loss such as switching loss of power devices. The switching vector selection and relative switch time calculation are realized by means of PWM voltage space vector algorithm. Another important thing is the neutral point control (NPC). The detailed contents of voltage space vector PWM algorithm with NPC could be seen in [1]-[2].

Complete *dq* decoupling control is implemented in a dual-DSPs board. In Fig. 4, 2# DSP can be used to fulfil the real-time algorithm of voltage space vector PWM. 1# DSP can be used to finish the decoupling control and DC voltage loop control as well as A/D synchronous sampling. Made use of dual-port RAM IDT7024, both digital signal processor TMS320c31 can operate parallel processing at 40MHz clock. The dual-DSPs board could also be used to finish other complicated control algorithms, such as AC motor control and active power filter, unbalanced control, voltage dynamic restore and other compensation functions using three-phase tri-level PWM converter.

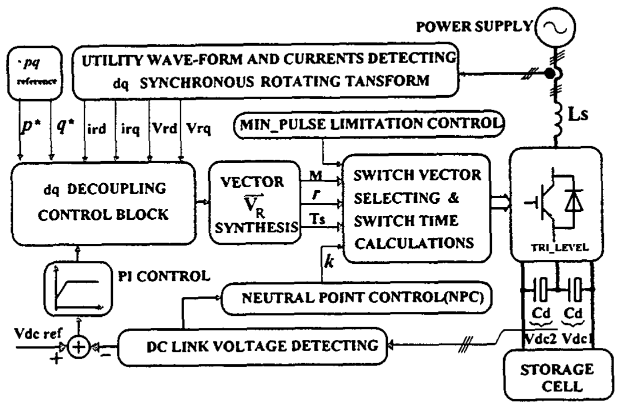

Fig. 3 The Control Block Diagram of the Unified DSP Control System

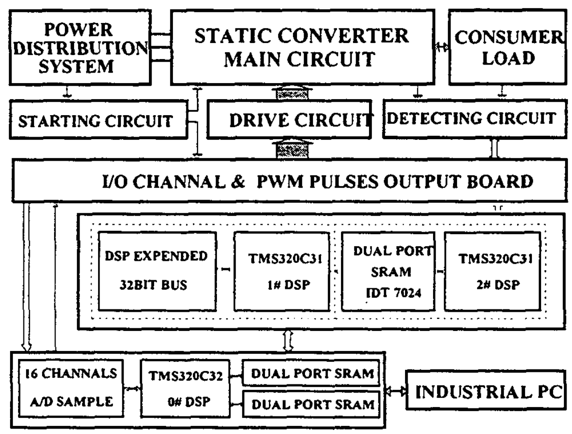

Fig. 4 The Control System of the Tri-level PWM Converter

IV. SIMULATION AND EXPERIMENTAL RESULTS

The computer dynamic simulation of the tri-level converter system was based on PSCAD® environment. The simulation was carried out under the following conditions: the three-phase AC power source voltage: V_{sm}=311V, ω=314.16, *f*=50Hz; the AC reactor: L_s=6mH, its equivalent R_s=0.5Ω; and DC load: C_d=4700μF, L_0=3mH.

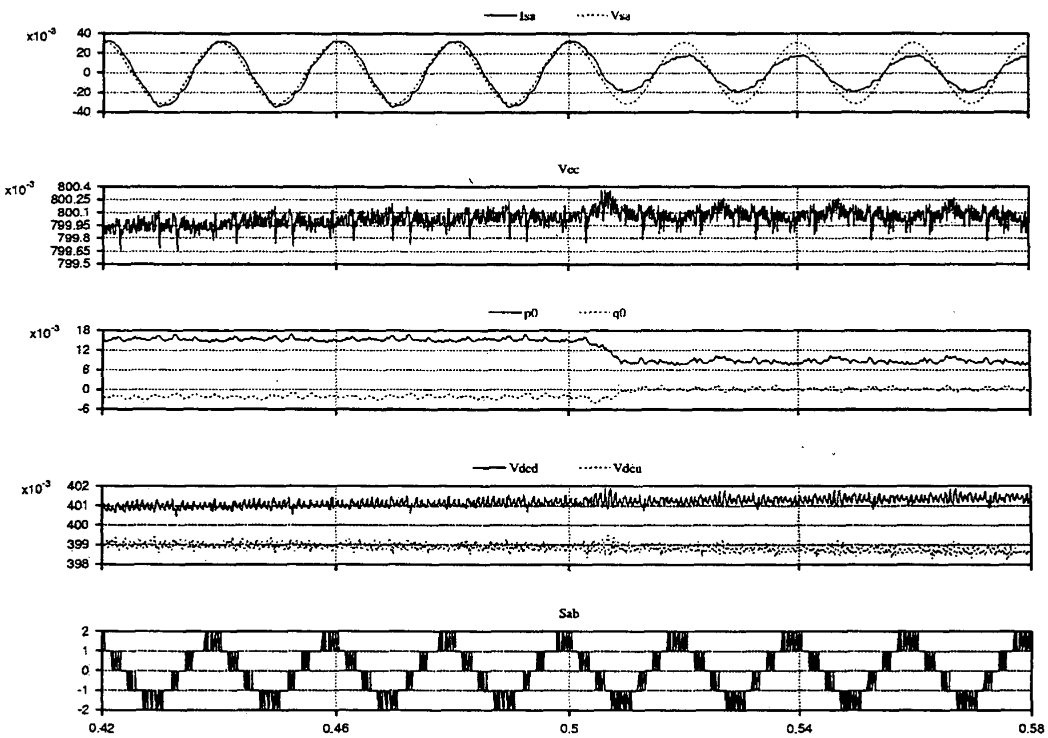

Fig. 5 The System Simulation of Three-phase Tri-level Converter

The minimum limitation of the ON pulse and OFF pulse width is $T_{pulse-\lim it} = 50\mu s$. The result of DC voltage close-loop control with P_{out}=15kW and V_{dcg}=800V and sampling frequency f_s=1200Hz is shown in Fig.5. The dynamic system response of active and reactive component step started at t=0.5025S. As shown in Fig. 5, the proposed system has attractive features such as controllable power factor, near sinusoidal current waveform, bi-directional power flow and dc bus voltage control. Moreover, the neutral point potential control makes up the balance of voltage between upper leg capacitor and lower leg capacitor.

A 10KW IGBT prototype of the tri-level reversible rectifier has been developed. The actual parameters of prototype are the same as those of computer simulation. In Fig. 6(a), the power factor has been controlled easily and here p.f. is equal to 1. In Fig. 6(b), the input currents i_{sA} and i_{sB} are nearly sinusoidal. As shown in Fig.6, the experimental results will verify the correctness of the proposed mathematical model of tri-level reversible rectifier system.

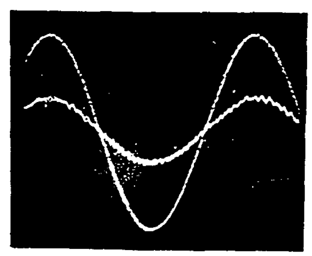

Fig.6 (a) Power Factor p.f. = 1 (the upper is V_{sA}, the lower is current i_{sA})

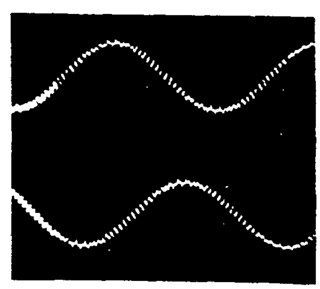

Fig.6(b) Waveform of Input Currents isA (upper) and isB (lower) (20A/div)

V. CONCLUSION

In this paper, a new mathematical model of tri_level PWM reversible rectifier was presented. The synchronous reference frame model was used to achieve the *dq* decoupling control of the converter. All control algorithms were implemented in dual digital-signal processors. The validity of the system model was verified by the results of computer-aided simulation and experiment.

REFERENCES

1. R. Rojas, T. Ohnishi, T. Suzuki, "An Improved Voltage Vector Control Method for Neutral-Point-Clamped Inverters ", IEEE Transaction on Power Electronics, Vol.10, No.6, 1995, pp:1156-1163.
2. Masato Koyama, et al., "Space Voltage vector-Based New PWM Method for Large Capacity Three-Level GTO Inverter", IEEE-PESC'92, pp:271-276.
3. Hyo L. Lin, Gyn H. Cho and Sun S. Pank, "Optimal PWM Design for High Power Three-Level Inverter Through Comparative Studies", IEEE Transaction on Power Electronics, Vol .10, No.1, 1995, pp:38-46.
4. J. B. Ekanayake, N. Jenkins, "A Three_Level Advanced Static VAR Compensator ", IEEE Transaction on Power Delivery, Vol.11, No.1, 1996, pp:540-545.
5. Zhan Changjiang, Study of the control techniques of system waveform of high power and high frequency PWM reversible rectifier, A Dissertation of Ph. D. 1997, Wuhan, P. R. China.