

## List of test functions in alphabetical order

There are 134 different test functions in total (each individual dimension of the same function is counted once). Different dimensions of the reference provided global minimizers [1,2] are separated by “|” and multi-solutions of minimizers under the same dimension are separated by “;”. The functions for which our algorithm was unsuccessful are marked with “\*”. Red indicates a problem of the results provided by Gavana [1] and Jamil and Yang [2] and based on our test results, in which the numbers (I)–(IV) correspond to the problematic categories.

Function numbers	Functions	Dimensions	Domain	Reference provided global minimum	Reference provided global minimizers
1	Ackley	2~5	[-32,32]	0	[0,0]   [0,0,0]   [0,0,0,0]  [0,0,0,0,0]
2	Adjiman	2	[-1,2,-1,1]	-2.02181	[2,0.10578]
3	*Alpine01	2~3	[-10,10]	0	[0,0]   [0,0,0]
4 (I)	Alpine02	2	[0,10]	-6.1295	[7.917,7.917]
5	BartelsConn	2	[-50,50]	1	[0,0]
6	Beale	2	[-10,10]	0	[3,0.5]
7	Bird	2	$[-2\pi,2\pi]$	-106.764537	[4.70104,3.15294;-1.58214,-3.13024]
8	Bohachevsky	2	[-100,100]	0	[0,0]
9	BoxBetts	3	[0.9,1.2;9,11.2;0.9,1.2]	0	[1,10,1]
10	Branin01	2	[-5,10;0,15]	0.397887357729 73816	[- $\pi$ ,12.275; $\pi$ ,2.275;9.42478,2.475]
11	Branin02	2	[-5,15]	5.559037	[-3.2,12.53]
12	Brent	2	[-10,10]	0	[-10,-10]
13	Brown	2~5	[-1,4]	0	[0,0]   [0,0,0]   [0,0,0,0]    [0,0,0,0,0]
14 (III)	Bukin02	2	[-15,-5;-3,3]	0	[-10,0]
15	Bukin04	2	[-15,-5;-3,3]	0	[-10,0]
16	Bukin06	2	[-15,-5;-3,3]	0	[-10,1]
17	CarronTable	2	[-10,10]	-24.1568155	[ $\pm$ 9.646157266348881, $\pm$ 9.646157266348881]
18	Chichinadze	2	[-30,30]	- 42.94438701899 098	[6.189866586965680,0.5]
19	Colville	4	[-10,10]	0	[1,1,1,1]
20 (III)	CosineMixture	2~10	[-1,1]	-0.2   -0.3   -0.4   -0.5   -0.6   -0.7   -0.8   -0.9   -1	[0,0]   [0,0,0]   [0,0,0,0]    [0,0,0,0,0]   [0,0,0,0,0,0]    [0,0,0,0,0,0,0]   [0,0,0,0,0,0,0,0]    [0,0,0,0,0,0,0,0,0]    [0,0,0,0,0,0,0,0,0,0]
21	CrossInTray	2	[-15,15]	- 2.062611870822 739	[ $\pm$ 1.349406608602084, $\pm$ 1.349406608602084]
22	*CrossLegTable	2	[-10,10]	-1	$x_1 = 0$ or $x_2 = 0$
23	Cube	2	[-10,10]	0	[1,1]
24 (III)	Deb01	2~5	[-1,1]	0	25 solutions   125 solutions   625 solutions   3125 solutions

25	Deb02	2	[0,1]	0	25 solutions
26	DeckkersAarts	2	[-20,20]	-24777	[0,±15]
27	*DeVilliersGlasser01	4	[1,100]	0	[60.137,1.371,3.112,1.761]
28	*DeVilliersGlasser02	5	[1,60]	0	[53.81,1.27,3.012,2.13,0.507]
29 (I)	DixonPrice	2~3	[-10,10]	0	$[1, 2^{-0.5}] \parallel [1, 2^{-0.5}, 2^{-0.75}]$
30 (III)	Dolan	5	[-100,100]	0.00001	$[8.39045925, 4.81424707, 7.3454133, 68.88246895, 3.85470806]$
31	Easom	2	[-100,100]	-1	$[\pi, \pi]$
32	EggCrate	2	[-5,5]	0	[0,0]
33	EggHolder	2	[-512,512]	-959.640662711	[512,404.2319]
34	ElAttarVidyasagarDutta	2	[-100,100]	1.712780354	[3.40918683,-2.17143304]
35 (I)	Exp2	2	[0,20]	0	$[1, 0.1]$
36	Exponential	2~3	[-1,1]	-1	[0,0]    [0,0,0]
37	FreudensteinRoth	2	[-10,10]	0	[5,4]
38	Giunta	2	[-1,1]	0.06447042053690566	[0.4673200277395354, 0.4673200169591304]
39	GoldsteinPrice	2	[-2,2]	3	[0,-1]
40	*Griewank	2	[-600,600]	0	[0,0]
41	Gulf	3	$[0.1, 100; 0.25, 6; 0.5]$	0	[50,25,1.5]
42 (III)	Hansen	2	[-10,10]	-2.3458	$[-7.58989583, -7.70831466]$
43	Hartmann3	3	[0,1]	-3.86278214782076	[0.1, 0.55592003, 0.85218259]
44	Hartmann6	6	[0,1]	-3.32236801141551	[0.20168952, 0.15001069, 0.47687398, 0.27533243, 0.31165162, 0.65730054]
45 (I)	*Himmelblau	2	[-6,6]	0	$[0, 0]$
46 (I)	HolderTable	2	[-10,10]	-19.20850256788675	$[\pm 9.664590028909654, \pm 9.664590028909654]$
47	Hosaki	2	[0,10]	-2.3458	[4,2]
48	JennrichSampson	2	[-1,1]	124.3621824	[0.257825, 0.257825]
49	Langermann	2	[0,10]	-5.1621259	[2.00299219, 1.006069]
50	Leon	2	[-1.2,1.2]	0	[1,1]
51	Matyas	2	[-10,10]	0	[0,0]
52	McCormick	2	[-1.5,4;-3,4]	-1.913222954981037	[-0.5471975602214493, -1.547197559268372]
53	MieleCantrell	4	[-1,1]	0	[0,1,1,1]
54 (II)	Mishra01	2~5	[0,1]	2	$[1, 1] \parallel [1, 1, 1] \parallel [1, 1, 1, 1] \parallel [1, 1, 1, 1, 1]$
55	Mishra02	2~8	[0,1]	2	$[1, 1] \parallel [1, 1, 1] \parallel [1, 1, 1, 1] \parallel [1, 1, 1, 1, 1] \parallel [1, 1, 1, 1, 1, 1] \parallel [1, 1, 1, 1, 1, 1, 1] \parallel [1, 1, 1, 1, 1, 1, 1, 1]$
56 (III)	Mishra03	2	[-10,10]	-0.18467	[-10,-10]
57 (I)	Mishra04	2	[-10,10]	-0.199409	[-10,-10]
58	Mishra05	2	[-10,10]	-0.119829	[-1.98682,-10]
59 (III)	Mishra06	2	[-10,10]	-2.28395	$[2.88631, 1.82326]$
60 (II)	*Mishra07	2~5	[-10,10]	0	$[\sqrt{2}, \sqrt{2}] \parallel [\sqrt{3}, \sqrt{3}] \parallel [2, 2] \parallel [\sqrt{5}, \sqrt{5}]$
61	Mishra08	2	[-10,10]	0	$[2, -3]$

62	Mishra09	3	[-10,10]	0	[1,2,3]
63	Mishra10	2	[-10,10]	0	[0,0;2,2]
64 (II)	Mishra11	2~5	[-10,10]	0	[0,0]   [0,0,0]   [0,0,0,0]   [0,0,0,0,0]
65	*Parsopoulos	2	[-5,5]	0	$[\pm 0.5\pi, 0]; [\pm 1.5\pi, 0]; [\pm 0.5\pi, \pm\pi]; [\pm 1.5\pi, \pm\pi]$
66	*Pathological	2	[-100,100]	0	[0,0]
67	Paviani	10	[2.001,9.999 ]	-45.7784684040686	[9.350266,9.350266,9.350266,9.350266,9.350266,9.350266,9.350266,9.350266,9.350266,9.350266]
68	Pinter	2~7	[-10,10]	0	[0,0]   [0,0,0]   [0,0,0,0]   [0,0,0,0,0]   [0,0,0,0,0,0]   [0,0,0,0,0,0,0]
69	Powell	4	[-4,5]	0	[0,0,0,0]
70	Price01	2	[-500,500]	0	[±5, ±5]
71	Price02	2	[-10,10]	0.9	[0,0]
72 (I)	Price03	2	[-50,50]	0	[±5, ±5]
73	*Price04	2	[-50,50]	0	[0,0;2,4;1.464,-2.506]
74	Qing	2~3	[-500,500]	0	$[\pm 1, \pm\sqrt{2}]   [\pm 1, \pm\sqrt{2}, \pm\sqrt{3}]$
75	Quadratic	2	[-10,10]	-3873.72418	[0.19388,0.48513]
76 (II)	*Quintic	2~3	[-10,10]	0	[-1,-1]   [-1,-1,-1]
77 (III)	Rana	2~3	[-500.000001, 500.000001]	-928.5478	[-500,-500]   [-500,-500,-500]
78	Ripple01	2	[0,1]	-2.2	[0.1,0.1]
79	Ripple25	2	[0,1]	-2	[0.1,0.1]
80	Rosenbrock	2~8	[-5,10]	0	[1,1]   [1,1,1]   [1,1,1,1]   [1,1,1,1,1]   [1,1,1,1,1,1]   [1,1,1,1,1,1,1]
81 (III)	RosenbrockModified	2	[-2,2]	34.37	[-0.9,-0.95]
82	RotatedEllipse01	2	[-500,500]	0	[0,0]
83	RotatedEllipse02	2	[-500,500]	0	[0,0]
84	Salomon	2	[-100,100]	0	[0,0]
85	Sargan	2~5	[-100,100]	0	[0,0]   [0,0,0]   [0,0,0,0]   [0,0,0,0,0]
86	Schaffer01	2	[-100,100]	0	[0,0]
87	Schaffer02	2	[-100,100]	0	[0,0]
88 (I)	*Schaffer03	2	[-100,100]	0.00156685	[0,1.253115]
89 (III)	Schaffer04	2	[-100,100]	0.292579	[0,1.253115]
90	Schwefel01	2~5	[-100,100]	0	[0,0]   [0,0,0]   [0,0,0,0]   [0,0,0,0,0]
91	Schwefel02	2~5	[-100,100]	0	[0,0]   [0,0,0]   [0,0,0,0]   [0,0,0,0,0]
92	Schwefel04	2~7	[0,10]	0	[1,1]   [1,1,1]   [1,1,1,1]   [1,1,1,1,1]   [1,1,1,1,1,1]   [1,1,1,1,1,1,1]
93	Schwefel06	2	[-100,100]	0	[1,3]
94	Schwefel20	2~4	[-100,100]	0	[0,0]   [0,0,0]   [0,0,0,0]
95	Schwefel21	2~4	[-100,100]	0	[0,0]   [0,0,0]   [0,0,0,0]
96	Schwefel22	2~6	[-100,100]	0	[0,0]   [0,0,0]   [0,0,0,0]   [0,0,0,0,0]   [0,0,0,0,0,0]
97	Schwefel36	2	[0,500]	-3456	[12,12]
98 (III)(IV)	Shekel05	4	[0,10]	-10.1527	[4,4,4,4]

99 (III)(IV)	Shekel07	4	[0,10]	-10.3999	[4,4,4,4]
100 (III) (IV)	Shekel10	4	[0,10]	-10.5319	[4,4,4,4]
101	Shubert01	2	[-10,10]	-186.7309	[-7.0835,4.8580](and many others)
102 (III)	Shubert03	2	[-10,10]	-24.062499	[5.791794,5.791794](and many others)
103 (I)(III)	Shubert04	2	[-10,10]	-29.016015	[-0.80032121,-7.08350592](and many others)
104 (I)(II)(III)	SineEnvelope	2~3	[-100,100]	0	[0,0]
105	SixHumpCamel	2	[-5,5]	-1.031628453489877	[0.08984201368301331,-0.7126564032704135]; -0.08984201368301331,0.7126564032704135]
106	Sphere	2~12	[-1,1]	0	[0,0]   [0,0,0]   [0,0,0,0]   [0,0,0,0,0]   [0,0,0,0,0,0]   [0,0,0,0,0,0,0]   [0,0,0,0,0,0,0,0]   [0,0,0,0,0,0,0,0,0]   [0,0,0,0,0,0,0,0,0,0]
107	StyblinskiTang	2~7	[-5,5]	-78.33233140754284   -117.49849711131426   -156.66466281508568   -195.8308285188571   -234.99699422262852   -274.16315992639994	[-2.903534018185960,-2.903534018185960]   [-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960]   [-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960]   [-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960]   [-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960]   [-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960]   [-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960,-2.903534018185960]
108 (II)	TestTubeHolder	2	[-10,10]	-10.872300	[-0.5 $\pi$ ,0]
109	ThreeHumpCamel	2	[-5,5]	0	[0,0]
110	Treccani	2	[-5,5]	0	[0,0;-2,0]
111	Trefethen	2	[-10,10]	-3.3068686474	[-0.02440307923,0.2106124261]
112	Trid	6	[-20,20]	-50	[6,10,12,12,10,6]
113	*Trigonometric01	2~5	[0, $\pi$ ]	0	[0,0]   [0,0,0]   [0,0,0,0]   [0,0,0,0,0]
114	*Trigonometric02	2~3	[-500,500]	1	[0.9,0.9]   [0.9,0.9,0.9]
115	Ursem01	2	[-2.5,3;-2,2]	-4.8168	[1.69714,0]
116	Ursem03	2	[-2,2;-1.5,1.5]	-3	[0,0]
117	Ursem04	2	[-2,2]	-1.5	[0,0]

118	UrsemWaves	2	$[-0.9, 1.2; -1.2, 1.2]$	-8.5536	[1.2, 1.2]
119	VenterSobieczczanski Sobieski	2	[-50, 50]	-400	[0, 0]
120 (I)	Watson	6	[-5, 5]	0.002288	[-0.0158, 1.012, -0.2329, 1.260, -1.513, 0.9928]
121	Wavy	2	$[-\pi, \pi]$	0	[0, 0]
122	WayburnSeader01	2	[-5, 5]	0	[1, 2]
123 (II)	*WayburnSeader02	2	[-500, 500]	0	[0.2, 1]
124 (I)	Weierstrass	2~7	[-0.5, 0.5]	4	[0, 0]   [0, 0, 0]   [0, 0, 0, 0]   [0, 0, 0, 0, 0]   [0, 0, 0, 0, 0, 0]
125	Whitley	2~12	$[-10.24, 10.24]$	0	[1, 1]   [1, 1, 1]   [1, 1, 1, 1]   [1, 1, 1, 1, 1]   [1, 1, 1, 1, 1, 1]   [1, 1, 1, 1, 1, 1, 1]   [1, 1, 1, 1, 1, 1, 1, 1]   [1, 1, 1, 1, 1, 1, 1, 1, 1]   [1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
126	Wolfe	3	[0, 2]	0	[0, 0, 0]
127	XinSheYang01	2~3	[-5, 5]	0	[0, 0]   [0, 0, 0]
128	XinSheYang02	2~3	$[-2\pi, 2\pi]$	0	[0, 0]   [0, 0, 0]
129	XinSheYang03	2~3	[-20, 20]	-1	[0, 0]   [0, 0, 0]
130	XinSheYang04	2~3	[-10, 10]	-1	[0, 0]   [0, 0, 0]
131	Zacharov	2~6	[-5, 10]	0	[0, 0]   [0, 0, 0]   [0, 0, 0, 0]   [0, 0, 0, 0, 0]   [0, 0, 0, 0, 0, 0]
132	Zettl	2	[-1, 5]	-0.0037912	[-0.029896, 0]
133 (IV)	Zimmerman	2	[0, 100]	0	[7, 2]
134	Zirilli	2	[-10, 10]	-0.3523	[-1.0465, 0]

### The problems of the tested functions.

- I. The minimum does not match the function value at the provided minimizer(s). This case can be verified by directly inputting the minimizer(s) into the function.
- II. The minimizers provided are incomplete. We found more minimizers and verified them too.
- III. Our algorithm found a strictly smaller minimum than the provided value [1, 2]. The new minimum is also verified.
- IV. The given formula is incorrect. We found this problem by comparing Gavana [1] to Adorio and Diliman [3].

### References:

1. A. Gavana, "Test function index" [http://infinity77.net/global\\_optimization/test\\_functions.html#test-functions-index](http://infinity77.net/global_optimization/test_functions.html#test-functions-index) (accessed 03/05/2021).
2. M. Jamil, X. S. Yang, A literature survey of benchmark functions for global optimisation problems. *Int. J. Math. Model. Numer. Optim.* **4**, 150-194 (2013).

3. E. P. Adorio, U. P. Diliman, MVF-Multivariate test functions library in C for unconstrained global optimization. <http://www.geocities.ws/eadorio/mvf.pdf> (2005). (accessed 03/05/2021).