Homework 1

1. Show that the trapezoidal method is absolutely stable using the example (on Page 30)

$$Y' = \lambda Y, \quad t > 0, \quad \lambda < 0$$
$$Y(0) = 1.$$

2. Show that the *Heun's method* (Page 40) is not absolutely stable, what's the region of absolute stability (Page 31)?

3. Derive the second-order Runge–Kutta methods (Page 56) corresponding to $b_2 = \frac{3}{4}$ and $b_2 = 1$. Please write the corresponding Butcher tableaus and the schemes. (Drawing an illustrative graph analogous to Page 57 helps you understand the scheme.)

4. Solve the problem (Page 58)

$$Y'(t) = -Y(t) + 2\cos(t), \qquad Y(0) = 1$$

with the formula $b_2 = 3/4$ from Problem 3. Present your codes and conduct the numerical tests with $h = 0.1$ and $h = 0.05$. Compare your results to those on Page 58.