1. Consider the following finite difference scheme for solving the two-point BVP $u''(x) = f(x)$, $a < x < b$, $u(a) = u_a$ and $u(b) = u_b$:

$$\frac{U_{i-1} - 2U_i + U_{i+1}}{h^2} = f(x_i), \quad i = 2, 3, \ldots, n-1,$$

where $x_i = a + ih$, $i = 0, 1, \ldots, n$, $h = (b-a)/n$. At $i = 1$, the finite difference scheme is

$$\frac{U_1 - 2U_2 + U_3}{h^2} = f(x_1).$$

(a) Find the local truncation errors of the finite difference scheme at x_i, $i = 2, 3, \ldots, n-1$, and x_1. Is this scheme consistent?

(b) Write the system matrix of this finite difference scheme. Is it symmetric? Is it diagonally dominant or weekly diagonally dominant?

2. Use the central finite difference scheme with the Ghost Point Method for solving the following PDE:

$$\begin{cases} u''(x) = f(x), \\ f(x) = -4\pi^2 \cos 2\pi x, \\ u(0) = 1, \\ u'(0.5) = 0. \end{cases}$$

The exact solution is $u(x) = \cos 2\pi x$. Verify the order of accuracy by using the infinite norm and the 2-norm (i.e., norm(U-u,inf)). Please plot the error norm and the slope of it into graphs as on Page 41 or 42 of the slide Chapter2.