

Homework 3

Modify the Matlab code `poisson_matlab.m` for Poisson equation to solve the general self-adjoint elliptic PDE

$$\begin{aligned}\nabla \cdot (p(x, y)\nabla u) - q(x, y)u &= f(x, y), \quad x \in (-\pi/2, \pi/2), \quad y \in (0, \pi), \\ p(x, y) &= x + y, \\ q(x, y) &= x^2 + y^2, \\ f(x, y) &= \cos x \cos y - \sin x \sin y - \cos x \sin y(x^2 + y^2) - 2 \cos x \sin y(x + y), \\ \frac{\partial u}{\partial x} &= \sin y, \quad \text{on } x = -\pi/2, \\ u &= 0, \quad \text{on } x = \pi/2, \\ u &= 0, \quad \text{on } y = 0, \\ u &= 0, \quad \text{on } y = \pi.\end{aligned}$$

Validate your code with analytic solution $u(x, y) = \cos x \sin y$. Analyze your numerical result and error. Plot the solution and the error.