1. Implement and test the upwind and the Lax–Wendroff schemes for the one-way wave equation

$$u_t + u_x = 0.$$

Assume the domain is $-1 \le x \le 1$, and $t_{final} = 1$. Test your code for the following parameters:

(a) $u(t,-1) = 0$, and $u(0,x) = (x+1)e^{-x/2}$.

(b) $u(t,-1) = 0$, and $u(0,x) = \begin{cases} 0 & \text{if } x < -1/2, \\ 1 & \text{if } -1 \le x \le 1/2, \\ 0 & \text{if } x > 1/2. \end{cases}$

Do the grid refinement analysis at $t_{final} = 1$ for case (a) where the exact solution is available, take $m = 10, 20, 40$, and 80. For problem (b), use $m = 40$. Plot the solution at $t_{final} = 1$ for both cases.

2. Use the upwind and Lax–Wendroff schemes for Burgers' equation

$$u_t + \left(\frac{u^2}{2}\right)_x = 0$$

with the same initial and boundary conditions as in problem 1.