Nonlocal Means Algorithm Using Superformula Kernel for Image Denoising

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***Abstract*—Using the superformula, a mathematic function describing many complex shapes and curves, this paper designs a new superformula kernel (SFK). We then introduce a novel nonlocal means (NLM) algorithm for image denoising by replacing the Gaussian kernel with the SFK. Simulations and comparisons demonstrate that the proposed kernel and algorithm show excellent denoising performance in terms of the peak signal and ratio (PSNR) and structural similarity (SSIM).**

***Index Terms*—Image denoising, Superformula kernel, Nonlocal Means, peak signal and ratio, structural similarity**

I. Introduction

Images are easily corrupted by noise during the image acquisition, transmission or other image processing stage. Noise removal for the corrupted images generally has a huge effect to subsequent image processing operations. Thus, image denoising plays a significant role in the preprocessing stage in many image processing tasks such as edge detection, image enhancement and segmentation, as well as pattern recognition [1].

Currently, the nonlocal means (NLM) algorithm as a patch-based denoising method has been widely used for removing the Gaussian noise from corrupted images. The basic idea is to compare dissimilarity between patches within the whole image and assign the higher weights to patches that are more similar. The NLM has been demonstrated to be a robust tool for image denoising. Many effects have done to improve its performance in different ways. These includes improving the dissimilarity measure criterion, weight estimation and parameter optimization. Deledalle *et al*. improved the NLM algorithm using the noise distribution model as a dissimilarity measure criterion and considering the weight assignment as a weighted maximum likelihood estimation (WMLE) problem [2]. In terms of weight estimation, Wu *et al*. [3] proposed a local James Stein type center pixel weights (LJSCPW) to improve the denoising performance of the nonlocal means. Besides Hua *et al*. made full use of the original information in method noise to improve the NLM in accuracy of the weight assignment [4]. Salmon considered the weight assignment as an optimization problem and improved the NLM using the Steins Unbiased Risk Estimate [5]. These methods intend to improve the weight assignment accuracy of the NLM using various techniques.

Different from these existing NLM methods, this paper works on the core of the NLM: the Gaussian kernel. Because the Gaussian kernel is a simple curve with two parameters, it may not be adaptive to complex noise circumstances and various requirements in real applications. Correspondingly, development of new kernels with robust structures becomes necessary.

Due to excellent properties of the superformula which contains several parameters and can produce many different curves [6, 7]. This paper proposes a new kernel named superformula kernel (SFK) which based on the superformula. Compared with classical kernels (i.e. Gaussian), the SFK is a complex but powerful kernel with more parameters and various distributions. Furthermore, we introduce an improved NLM by replacing the Gaussian kernel with the superformula kernel. Simulation results are provided.

This paper is organized as follows: Section II briefly reviews the classical NLM algorithm and superformula as a background. Section III proposes the superformula kernel and a new nonlocal means algorithm based on this kernel. Simulations and comparisons are presented in Section IV and Section V concludes this paper.

II. Preliminary

A. Nonlocal means algorithm

The noisy image model, which describes an image corrupted by an additive white Gaussian noise (AWGN), can be defined as

$$y_{i,j} = x_{i,j} + n_{i,j},\ and\ n_{i,j} \sim \aleph(0, \sigma^2) \tag{1}$$

where $x_{i,j}$ is the clean image, $y_{i,j}$ is the noisy image, $n_{i,j}$ is noise with normal random variables and variance σ^2 with *zero* mean. Therefore, the denoised image $\widetilde{x}$ using the NLM algorithm is defined by:

$$\widetilde{x}_{i,j} = \frac{\sum w_{i,j} y_{i,j}}{\sum w_{i,j}} \tag{2}$$

where the weight $w_{i,j}$ is computed using the noisy patches $y_{i,j}$ centered at pixel (i, j) and reference patches $\widetilde{y}$ in the noisy image. The weight $w_{i,j}$ can be defined as:

$$w_{i,j} = exp(-\frac{1}{h^2}\|y_{i,j} - \widetilde{y}\|_2^2) \tag{3}$$

where $h = 10\sigma$ is the smoothing parameter to control behaviors of the weight function. Parameter σ is the noise

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level in a noisy image. $\|\cdot\|_2^2$ is the Euclidean norm which measures the dissimilarity between patches.

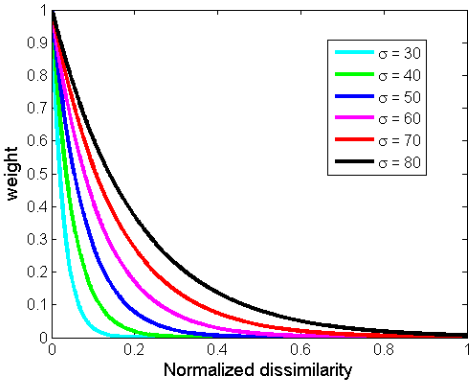


Fig. 1. Weight distribution using the Gaussian kernel under different noise levels σ.

Fig. 1 shows the weight distribution of the Gaussian kernel with different noise levels. Here we normalize the dissimilarity between two patches within [0, 1]. Weight $w_{i,j}$ is shown in the y axis with the maximum value of 1. We can see that the curves of the Gaussian kernel converges to 0 with the converging rate related to the noise level. These simple weight distributions of the Gaussian kernel may not be suitable for complex situations and requirements in practical applications. In the next section, we will design a new kernel to overcome this drawback.

B. Superformula

The superformula proposed by Gielis can be represented as [6]:

$$r(\theta) = \left(\left| \frac{cos\left(\frac{m\theta}{4}\right)}{a} \right|^{n_2} + \left| \frac{sin\left(\frac{m\theta}{4}\right)}{b} \right|^{n_3} \right)^{-\frac{1}{n_1}} \quad (4)$$

where parameters are the radius r, angle θ, rotational argument m, exponents n_i and the short and long axes a and b where $a, b \neq 0$.

The superformula in Equation (4) is in the polar coordinate plane. It can be transformed into the Cartesian coordinate plane by [7]:

$$\begin{cases} x = r(\theta)\, cos\theta \\ y = r(\theta)\, sin\theta \end{cases} \quad (5)$$

where x and y are coordinate values in the x and y axes, respectively.

The superformula describes two-dimensional (2D) shapes and curves in the polar coordinate plane. Obviously, changing its parameters (m, n_i, a, b) will create a large number of various shapes and curves. When a and b are different, the superformula generates shapes closer to ellipses in which the long and short axes are not symmetry; Otherwise, it creates isotropic shapes. This indicates that the superformula was derived from the function of an ellipse or a circle.

Due to the excellent property of the superformula in generating many complex shapes and curves, this paper investigates its applications in image processing. Particularly, we will use it to design a new image filtering kernel.

III. Nonlocal means algorithm using superformula

This section first proposes a new superformula kernel and introduces a novel nonlocal means algorithm using this kernel.

A. Superformula kernel

The NLM uses the Gaussian kernel to compute the weight of each pixel. For a given noise level σ and slightly variations in $\|y_{i,j} - y\|$ of two specific patches, the weight calculated from the Gaussian kernel will be always fixed. In practical applications, users may want to have the capability to change the weight in such a situation to achieve the best denoising performance. Hence we propose a new superformula kernel. It will be used for our NLM algorithm.

As shown in Equation (5), both x and y are controlled by θ. The mathematic relationship between x and y is a one-to-one mapping via a specific θ. Thus we define a general formula called the superformula kernel (SFK), which presented by

$$y_k = SF(x_k) \quad (6)$$

where SF stands for the superformula, x_k and y_k denote the input and output of the SFK, respectively.

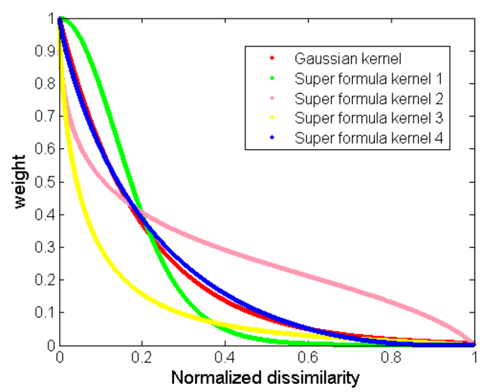


Fig. 2. Comparison of the Gaussian kernel and superformula kernel in terms of the weight distribution, under the same noise level, $\sigma = 80$.

Fig. 2 compares the weight distributions between the SFK and the Gaussian kernel. As can be seen, for a specific noise level, i.e. $\sigma = 80$, the weight distribution of the Gaussian kernel is fixed as shown as the red curve in Fig. 2. However, the SFK is able to provide various weight distribution schemes based on different parameter settings. The blue, green, yellow and pink curves in Fig. 2 are four examples of the SFK weight distributions. This offers the proposed SFK the design flexibility to meet various or special denoising requirements in real-world applications.

B. Proposed Algorithm

By replacing the Gaussian kernel with the superformula kernel in Equation (6), we propose a new nonlocal means algorithm, called the nonlocal superformula (NLSF) algorithm. The denoising model can still use Equation (2), but the weight is defined by,

$$w_{i,j} = SF(\|y_{i,j} - \widetilde{y}\|_2^2) \tag{7}$$

Changing the parameter setting of the superformula in Equations (4) and (5), we can obtain a different weight assignment scheme for the proposed NLSF.

IV. Experimental Results

Our proposed NLSF has been successfully applied to various images. This section provides several simulation results to show its denoising performance.

To quantitatively assess the denoising performance, we use the structural similarity index (SSIM) [8] and peak signal-to-noise ratio (PSNR) [9] to measure the denoised results. The PSNR is defined as :

$$PSNR = 10 \cdot \log_{10}\left(\frac{MAX_I^2}{MSE}\right) \tag{8}$$

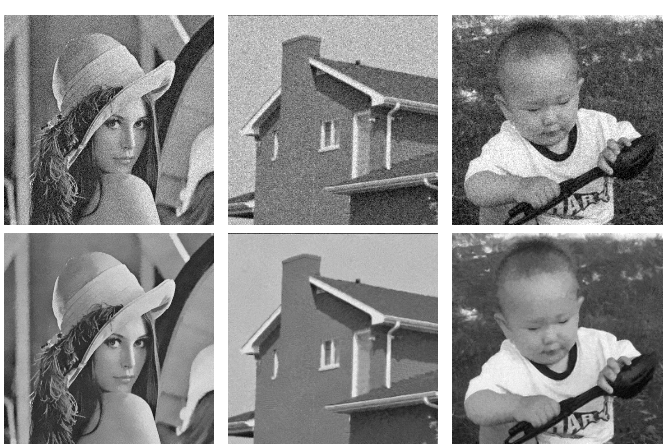
where MAX is the maximum value of an image; MSE is the average of the pixel-to-pixel differences between two images. It is defined as:

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(y_{i,j} - x_{i,j})^2 \tag{9}$$

where x and y are the noise-free and noisy images with size of $M * N$, respectively.

A. Denoising performance

We first test the denoising performance of the NLSF on different images. The results are shown in Fig. 3. We add the $\sigma = 40$ Gaussian noise to three grayscale images and then apply the NLSF to images.



(a) (b) (c)

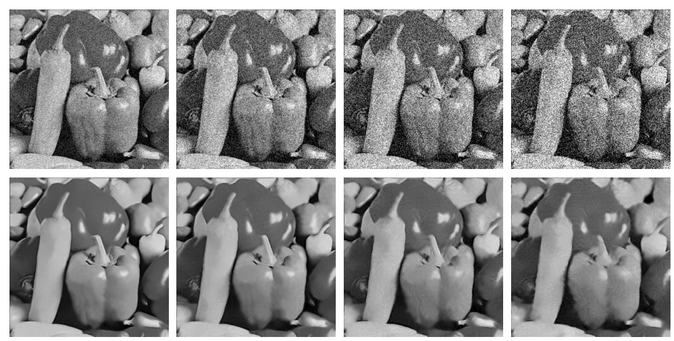
Fig. 3. Image denoising of the NLSF on different images. The top row shows noisy images with additive Gaussian noise, $\sigma = 40$; the bottom row shows the denoised results by the NLSF. (a) Lena; (b) house; (c) child.

TABLE I
THE PSNR MEASURES OF THE NOISY AND DENOISED IMAGES IN FIG. 3

image	**Lena**	**House**	**Child**
Noisy Image	24.7547	24.4144	24.4027
NLSF	26.0189	28.4148	26.1947

As shown in Fig. 3, noise in these noisy images are removed while preserving image details. In Fig. 3(a), the background of *Lena* is smoothing and clear. In image Fig. 3(b), the denoised image preserves edges and corners. In Fig. 3(c), the denoised image have good visual quality. Table I gives the PSNR measure results of images before and after denoising. All images denoised by the NLSF have higher PSNR values than their corresponding noisy images.

We then test the denoising performance of the NLSF when noise levels change. The denoising results are shown in Fig. 4. Table II provides their PSNR measure results. As can be seen, all denoised images have good visual quality and have higher PSNR values than their noisy images. These demonstrate the NLSF's excellent denoising performance.



(a) (b) (c) (d)

Fig. 4. Image denoising using the NLSF on different noise levels. The top row shows noisy images with different noise levels; the bottom row shows the dnoised results; (a) $\sigma = 20$; (b) $\sigma = 30$; (c) $\sigma = 40$; (d) $\sigma = 50$.

TABLE II
THE PSNR MEASURES OF IMAGE DENOISING USING THE NLSF WITH VARIOUS NOISE LEVELS

level	$\sigma = 20$	$\sigma = 30$	$\sigma = 40$	$\sigma = 50$
Noisy Image	29.9329	26.3792	24.1216	23.2164
NLSF	30.1806	28.1353	25.8516	24.2669

B. Performance comparison

We here compare the proposed NLSF with the classical nonlocal means algorithm. In our experiments, we set the patch size $7 * 7$ and the searching window $21 * 21$, which are the same settings as the literature in [1]. Without loss of generality, we list the PSNR and SSIM of denoising results on several benchmark images from USC-SIPI[1] in Table III, where values in the bold fonts indicate the better denoising performance. As can be seen, both PSNR and SSIM results show that our

[1]The USC-SIPI database is located in: http://sipi.usc.edu/database/

TABLE III
COMPARISON OF THE NLM AND NLSF WITH PSNR AND SSIM AT DIFFERENT NOISE LEVELS

Image	Method	PSNR							
		$\sigma = 10$	$\sigma = 20$	$\sigma = 30$	$\sigma = 40$	$\sigma = 50$	$\sigma = 60$	$\sigma = 70$	$\sigma = 80$
Cameraman	NLM	29.5387	26.6358	24.2283	22.7205	21.7788	21.1162	20.6826	20.1528
	NLSF	**30.7726**	**28.6946**	**27.2286**	**25.2898**	**23.8149**	**22.1595**	**22.1595**	**21.5011**
Lena	NLM	31.0875	27.2372	24.9960	23.6510	23.6052	22.0210	21.5050	21.0752
	NLSF	**33.2546**	**29.9951**	**27.8542**	**26.0189**	**24.5048**	**23.0774**	**22.6603**	**22.1013**
Pepper	NLM	31.5914	27.2802	24.8528	23.2701	22.2075	21.5340	20.9586	20.5269
	NLSF	**33.3778**	**30.1806**	**28.1353**	**25.8516**	**24.2669**	**22.6941**	**22.3629**	**21.7872**
House	NLM	**33.7724**	29.4340	26.2951	24.5920	23.5308	22.8410	22.3220	21.9737
	NLSF	33.3778	**30.1806**	**28.4148**	**28.4148**	**25.8523**	**23.9280**	**23.4483**	**22.9801**
		SSIM							
Cameraman	NLM	83.41	77.91	73.39	69.51	66.70	63.99	62.00	59.51
	NLSF	**88.06**	**81.00**	**74.12**	**70.92**	**67.11**	**64.81**	**64.81**	**61.69**
Lena	NLM	87.36	79.34	73.82	69.72	66.04	63.41	61.27	59.07
	NLSF	**91.54**	**84.56**	**76.72**	**72.71**	**68.04**	**65.12**	**63.57**	**6090**
Pepper	NLM	88.82	81.78	76.57	72.23	69.08	66.16	63.89	61.35
	NLSF	**91.48**	**85.45**	**78.24**	**74.63**	**70.89**	**67.75**	**66.22**	**63.43**
House	NLM	86.18	81.17	77.01	73.90	71.10	68.87	66.55	64.81
	NLSF	**88.95**	**83.69**	**79.26**	**79.26**	**71.83**	**69.03**	**66.96**	**65.04**

NLSF outperforms the classical nonlocal means algorithm in all test images with most noise levels, particularly in the heavy noise levels (σ is greater than 60).

V. CONCLUSION

To investigate the application of the superformula in image processing, this paper has proposed a new superformula kernel. This kernel has several parameters and offers users the design flexibility to meet complex requirements in practical applications. By replacing the Gaussian kernel with this kernel, we have introduced a new nonlocal means algorithm. Image denosing results and quantitative measures have demonstrated the excellent denoising performance of the proposed kernel and algorithm.

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