Non-harmonic Fourier transforms

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Abstract

Motivated by the non-harmonic Fourier series which originated from the celebrated work of Paley and Wiener, we introduce a integral version of the non-harmonic Fourier series, called the non-harmonic Fourier transform, denoted by $\mathcal{F}_a$. It is a bijection in $L^2(\mathbb{R})$ and is an integral transformation with respect to the harmonic measure $d\theta_a$, taking the non-linear Fourier atom $e^{i\theta_a(t)\theta_a(\omega)}$ as a reproducing kernel, where θ_a is the argument function of the boundary value of Möbius transformation in the unit disc. It turns out that this nonharmonic Fourier transform $\mathcal{F}_a$ is a unitary transform in $L^2(\mathbb{R}, d\theta_a)$ and it can be explicitly defined in terms of generalized Hermite polynomials. The discrete version of the transform and the corresponding results on the Shannon sampling theorem and the Poisson summation formula are also considered. The new phenomena encountered with the non-harmonic Fourier transform is that the sampling points are non-equally distributed. Since the nonharmonic Fourier transform interchanges weighted derivatives into multiplications, it plays a role in solving certain differential equations with variable coefficients. This is a joint work with Paula Cerejeiras, Qiuhui Chen, and Uwe Kähler.

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